Processing Equipments Design

3. Equipments dimensioning
Economical utilization of material
Plasticity

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Equipment dimensioning
Economical utilization of material

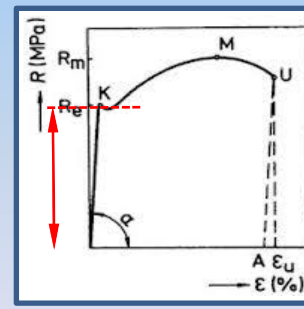
1. Original method – ideally elastic material:

Equipment or structures dimensioning whose material is ideally elastic and the stress is lower than the yield point (the range of Hooke´s law validity = linear dependence of stress on elongation).

Maximal stress in the material must be lower than is the yield point!

- **Advantage:** Simple calculations

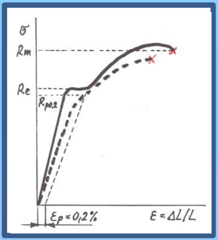
$\mathrm{tg}\,\alpha = E$
$\sigma = E*\varepsilon$
$\varepsilon = \Delta L/L$



- **Disadvantage:** Such designed structure has a reserve
It is able to withstand higher load (in some cases!)

2. Method taking into account elastic & plastic deformations of material:

What is a behavior of a structure (equipment) subject to the elastic and plastic deformations → the structure load increasing?



- **External loading causes firstly elastic deformation.**
- **When the load in some structure places is higher (places with the highest stress) plastic deformations start to form there ($\sigma_{local} = \sigma_Y$).**
- **The yet higher load causes that the number of these places and their extent escalates. Final state is the structure collapse (rupture of the most loaded part, total collapse or too big deformations).**
- **The state of stress in the structure just before the collapse has name „ limit state (extreme state, stress limit)“.**

For structures, equipment and parts appreciation from the point of view of their operational reliability and safety following types of limit states are used:

- **Elastic limit state** – when is the load higher than the value, first plastic deformations start to form somewhere in a structure (old classic way)
- **Strength limit state** – when a material consistency is reached a part is broken (local or total) or a fracture rises (brittle failure, plastic failure, ductile, mixed, fatigue fracture or creep fracture)

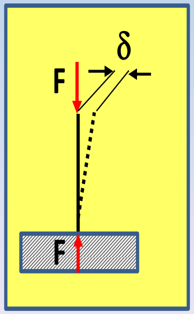
  → a rupture of a wall, beam breaks, cracks in steet ...
- **Deformation limit state** – when it is reached structure starts to have non-permissible deformations

  (structure usually doesn´t break but it bends)
- **Load limit state** – when it is reached a structure with firm shape loses its connections and changes into vague structure or mechanism (structure connections are lost by a failure (rupture, break) of some structure part or parts – types of failures see above)

- **Limit state of adaptation** – till the state elastic deformations do not exceed some value even if the overloading is repeated → this situation is used in practice → **better material utilization = modern way of a structure design** = It will be discussed in the following part of this lecture

  → a structure is adapted to the 1st overloading (in all profile or in some parts = stress peaks)

- **Limit state of stability** – when it exceeds non-permissible deformations and/or failures rise very quickly (beam buckling, cylinder loaded with external overpressure ... → sudden change of a structure shape – more in the part 8.)

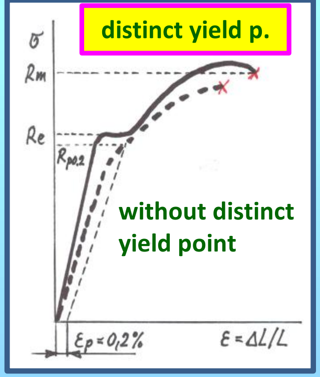


- The theory of limit states is applicable for tough materials with marked yield point σ_Y.

EN tough D zäh IT tenace, duttile FR tenace ESP tenaz, ductil
RUS вязкий материал

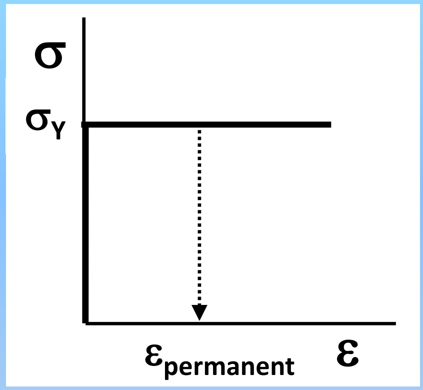
3. Basic models of tough materials:

Real situation



1. Ideally plastic

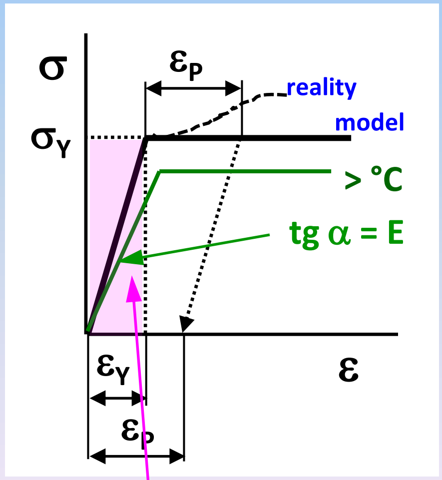
Theoretical material that does not exist in praxis (till the σ_Y is reached there is no deformation → $E = \infty$; for higher load it has only plastic deformation → Young´s modulus $E = 0$)



2. Ideally elastic-plastic

Models

A presumption is, that till reaching the yield point material behaves ideally elastic ($\sigma = E * \varepsilon$). When the yield point is reached the stress is constant ($= \sigma_Y$) but deformation rises → $E = 0$



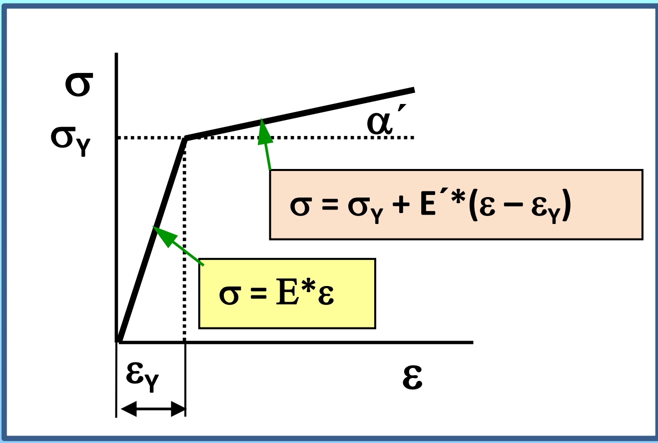
no permanent deformation

region of Hooke´s law validity

The model is often used for engineering calculations

3. With linear hardening

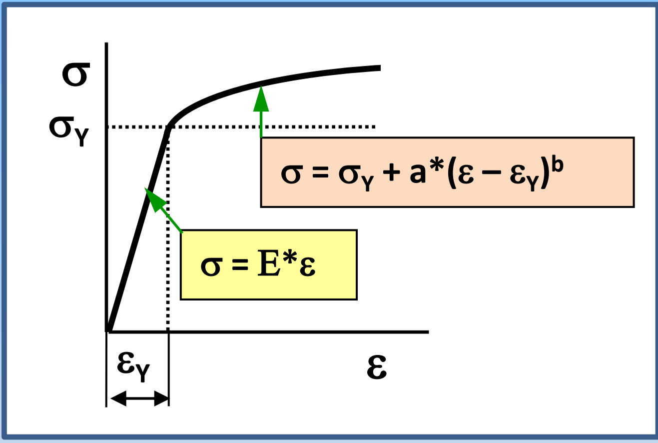
When yield point is exceeded the material is harder (it withstands higher stress). Dependence of the hardening on extension is done by a line that slope $\alpha´$ is defined by the modulus of hardening $E´$.



Models

4. With exponential hardening

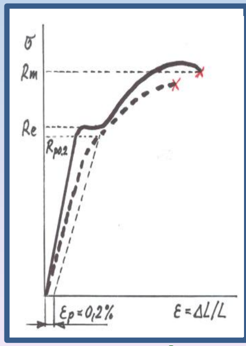
When yield point is exceeded the material is harder according exponential dependence.



These models better conform with the real stress curve

As $E´ << E$ difference of results of calculations according models 3. or 4. compared to model 2. are negligible

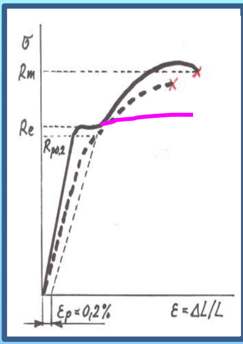
Real situation



In engineering praxis ideally elastic-plastic model is used.

We calculate that a material can withstand σ_Y, but it is able to withstand higher stress (mat. hardening)

- **For pressure vessels steels with low carbon content are used. Such steels have very low hardening. For such steels is the model valid. The simplification is on the side of higher safety (calculated maximum permitted load is a bit lower than the real load with the hardening).**



- **Models 3. a 4. are used for more exacting calculations with using of numerical methods (e.g. for very expensive materials – they allow higher material load compared the elastic-plastic model).**

- **On the material plasticity temperature and time have effect too (time of external loading, its uniformity or variation).**

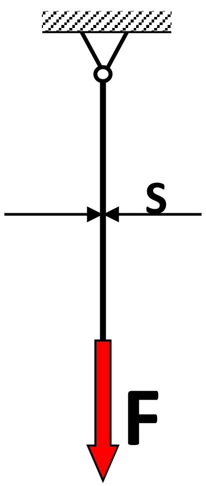
- **In praxis in structures and/or apparatuses an additional internal stress exists (effect of welding, thermal treatment, forming, cutting, mounting etc.). It can worsen strength conditions of such structure (but sometimes for example a pre-stress can improve conditions in structure – see later).**

4. Economical utilization of material plasticity

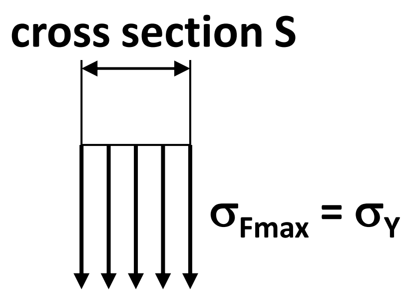
For all examples in the chapter we suppose an ideally elastic-plastic model (it is without any hardening).

4.0 Plasticity for uni-axial tensile load of bar

Testing of the simplest example = a tensile load of a beam

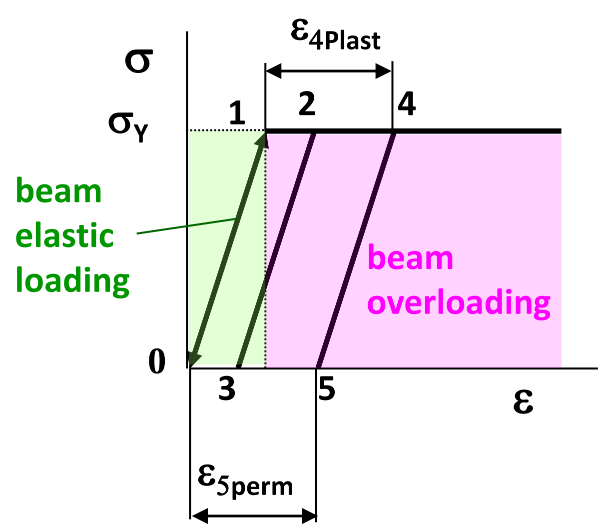


beam loaded with axial force



$$\sigma_F = F / S \leq \sigma_Y$$

$\rightarrow$ all parts of the profile have the same stress



With every next overloading the bar starts to be longer and longer $\rightarrow$ fatigue failure

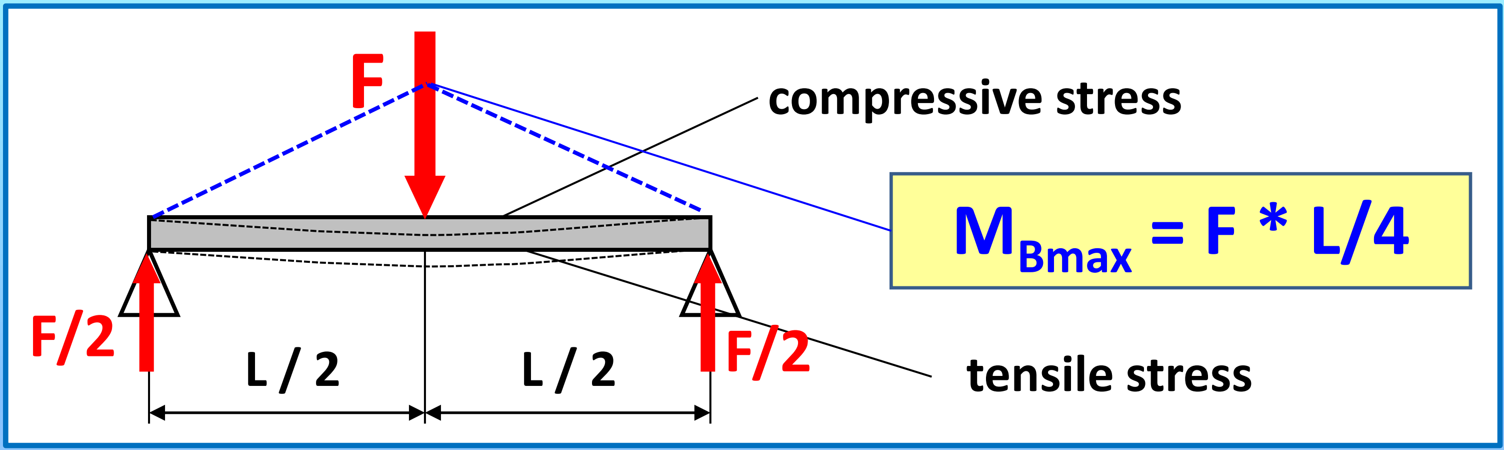
Steps of the bar loading, overloading and unloading:

0 – 1 – 0	$F < \sigma_Y * S$	only elastic deformations (after unloading is $\varepsilon_{perm} = 0$)
1 – 2	$F \geq \sigma_Y * S$	plastic deformation ε_{perm} = 1-2 = 0-3
2 – 3	unloading	after unloading bar has permanent deformation ε_{perm} = 0-3
3 – 2 – 3	$F < \sigma_Y * S$	only elastic deformations (after unloading ε_{perm} = 0-3)
3 – 2 – 4	$F \geq \sigma_Y * S$	new plastic deformation ε_{perm} = 2-4
4 – 5	unloading	after unloading the bar has permanent deformation ε_{perm} = 0-5
etc.		

In the case all material "strings" (parts) are plasticized and all lengthen in one time. When is the overloading and unloading repeated a fatigue failure comes →

It is impossible to overload such loaded bar (or structure) repeatedly!!

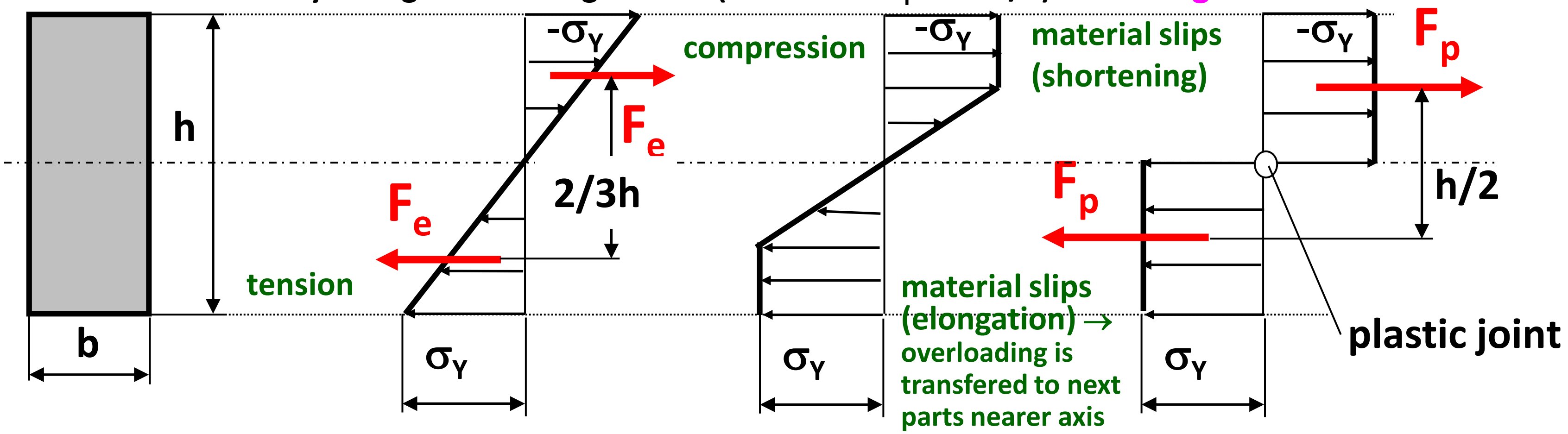
4.1. Example of loading of rectangular beam with section b x h with bending force



- By gradual increasing of the beam loading in its profile stress rises. The stress increases till it reaches in outer strings the yield point $\pm\sigma_Y$.

  It is the maximal beam load according the classic method.
- But the capacity of the beam according the theory of limit states is not fully utilized. For higher loading parts of the beam profile starts to plasticize gradually from outer parts towards to the center.
- When is the all profile plasticized in the axis so-called plastic joint arises. Originally triangular stress profile changes into a rectangular profile.

Force acts in a gravity center of the area. For triangle is the gravity center in 2/3 of leg, for rectangle in 1/2.
Force size is determined by triangle or rectangle areas (with sides σ_Y and h/2). Bending moment = forces x forces arm



Beam profile	Elastic deformation	Part. plastic deformation	Fully plasticized beam profile

Maximal bending moment in elastic region

F_e - max. force for elastic profile loading

$$M_{emax} = F_e * arm = 1/2 * h/2 * \sigma_Y * 2/3h = 1/6 * h^2 * \sigma_Y$$

Maximal bending moment for fully plasticized profile

F_P - max. force for fully plasticized profile

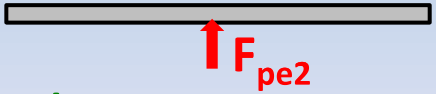
$$M_{pmax} = F_p * arm = h/2 * \sigma_Y * h/2 = 1/4 * h^2 * \sigma_Y$$

Coefficient of plasticity C_p is ratio M_{pmax} / M_{emax} .

$$C_p = (1/4 * h^2 * \sigma_Y) / (1/6 * h^2 * \sigma_Y) = 6 / 4 = 1.5$$

It follows from it that for a rectangular beam profile is the maximal plastic bending moment 1.5 times higher than the maximal elastic bending moment. Therefore it is possible to increase the bending loading of the beam 1.5 times or decrease the safety factor (for example from value $x = 1.5$ to $x = 1$).

When such fully plasticized profile is unloaded, in outer profile strings stress arises (as strings are elongated (down) or shortened (up)). The stress corresponds to an opposite bending moment with value M_{pe2} (= beam pre-loading with this elastic moment).



$$M_{pe2} = (1 - C_{e2}) * M_e$$

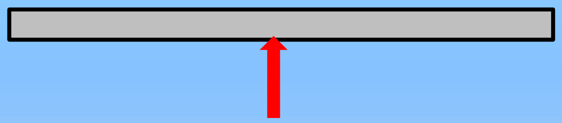
(only for information)

$C_{e2} = 2/C_p - 1$ coef. of residual deformation after plastic joint arise

$C_{e2} = 2/1.5 - 1 = 0.33$ (for rectangular profile it is $C_{e2} = 0.33$)   (→ after unloading is in outer strings reserve 33% to the σ_Y)

For some profiles are values of C_p and C_{e2} in following table.

- For profiles with $C_p = 2$ after unloading in outer strings arise stress with value equals to the yield point with opposite sign (e.g. instead tensile is compression).



- For profiles with $C_p > 2$ the opposite plastic deformation arises after unloading in outer strings too $\rightarrow$ danger of fatigue loading.

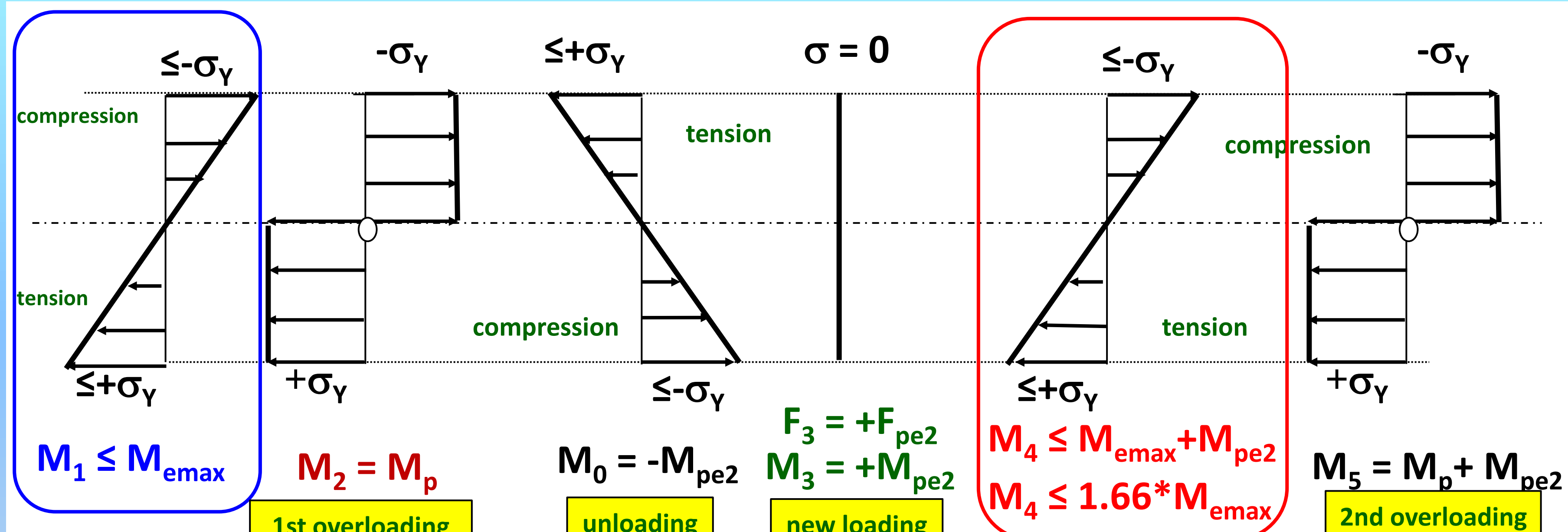
Ex.: $C_p = 1.5$; $C_{e2} = 0.33 \rightarrow$ $M_{pe2} = -0.666 * M_e$

$C_p = 2.0$; $C_{e2} = 0.00 \rightarrow$ $M_{pe2} = -1.00 * M_e = M_{elastmax}$

$C_p = 2.2$; $C_{e2} = -0.10 \rightarrow$ $M_{pe2} = -1.10 * M_e > M_{elastmax}$

coef. of plasticity — coef. of residual deformation — bending moment after unloading

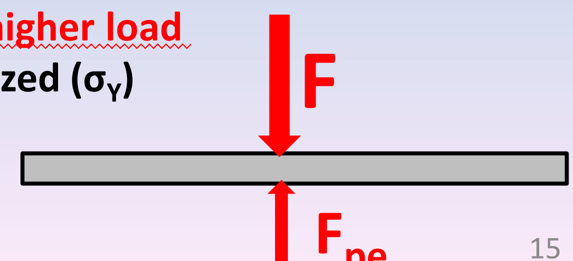
Example of alternating loading, plasticizing, unloading and new loading of a beam.



loading in elastic region

→ the beam is adapted to the 1st overloading

- M_1 loading with such moment, when in surface strings is stress σ_Y; → = max. elastic load
- M_2 loading with such moment, that the profile is fully plasticized; → = max. plastic load
- M_0 profile unloading; owing to permanent deformations an elastic stress arises ($\leq \sigma_Y$);
- M_3 loading with such moment when in the profile is not any stress;
- M_4 loading with such moment, when in surface strings is stress σ_Y ($M_4 > M_1$); = new max. elastic load
  → such pre-stressed beam is able to withstand in the elastic state higher load
- M_5 loading with such moment, that the profile is again fully plasticized (σ_Y)
  → danger situation (fatigue loading)

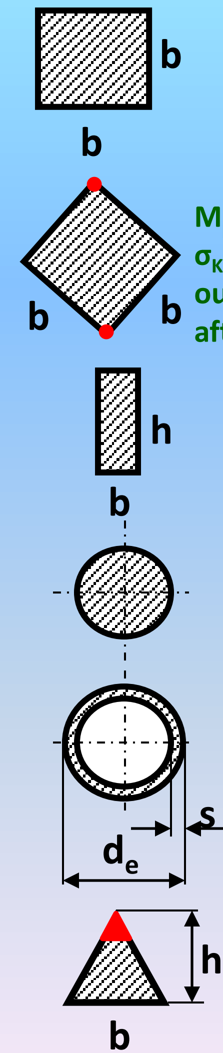


Fatigue loading results is fatigue failure.

Tab.1 Values of elastic and plastic section modulus W_e and W_p, coefficient of plasticity C_p and coefficient of residual deformation C_{e2} for various profiles

$C_{e2} = 2/C_p - 1$ reserve to reach the yield stress after a profile unloading

Profile	W_e	$W_p = C_p{*}W_e$	C_p	C_{e2}
Square (side b)	$1/6b^3$	$1/4b^3$	1.5	0.33
Square skew (side b)	$b^3/(6{*}\sqrt{2})$	$\sqrt{2}{*}b^3/6$	2.0	0.0
Rectangular (height h, side b)	$1/6bh^2$	$1/4bh^2$	1.5	0.33
Circle (diameter d)	$\pi d^3/32$	$d^3/6$	1.7	0.176
Annulus (diameter d_e, wall thickness s)	$0.78{*}d_e^2{*}s$	$d_e^2{*}s$	1.27	0.575
Triangle (base b, height h)	$bh^2/12$	$2{*}\sqrt{2}/6{*}bh^2$	2.34	-0.145



$M_{pe2} = M_e \rightarrow \sigma_K$ is reached in outer strings after unloading

$M_{pe2} = (1 - C_{e2}) * M_e$

$M_{pe2} = -1.145{*}M_{emax}$ $\rightarrow \sigma_K$ is reached in more strings after unloading

$\rightarrow$ danger of fatigue failure

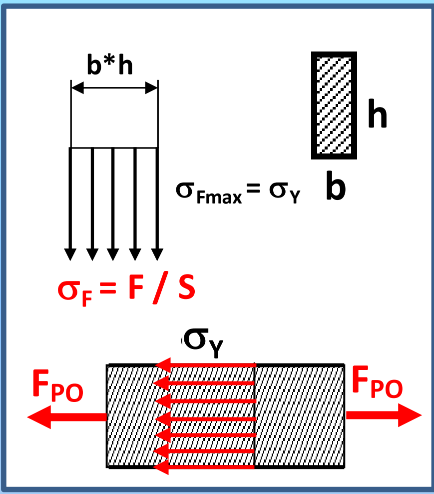
4.2. Plasticization for combine load (tensioning + bending)

Firstly we suppose that the beam is loaded only with an axial force F_P

Limiting force for loading with only force F_{PO} *(maximal stress is σ_Y)*

$$F_{PO} = h * b * \sigma_Y$$

(area on what acts tension times max. tension σ_Y)



Secondly we suppose that the beam is loaded only with a moment M_P

Limiting moment for loading with only moment M_{PO}

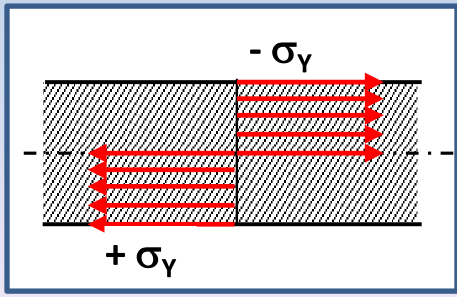
(max. loading for fully plasticized profile)

$$M_{PO} = W_p * \sigma_K = W_o * C_p * \sigma_Y = 1/4 * b * h^2 * \sigma_Y$$

$$(=1/6 * b * h^2 * 1{,}5 * \sigma_Y)$$

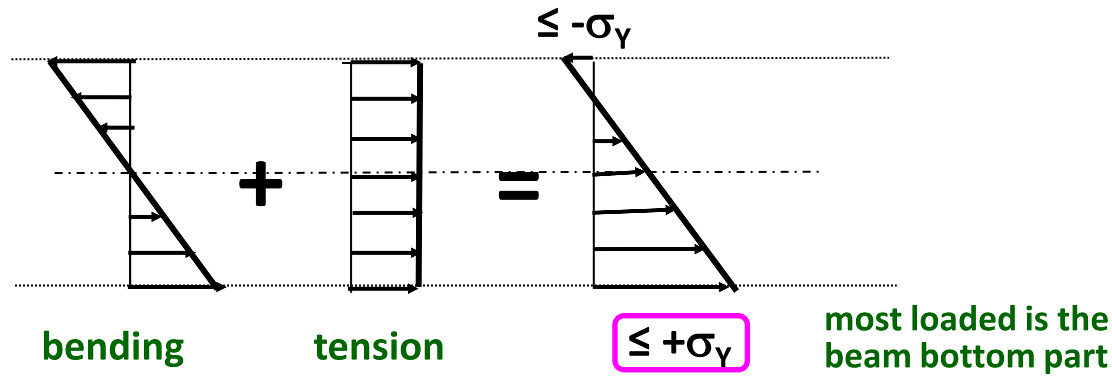
(see above the profile for the bend)

The stress profile in the beam for the combine loading under limiting state is on the following fig. (superposition of both loadings has to be on the yield point = limiting state – maximal total load is such that all profile is plasticized).

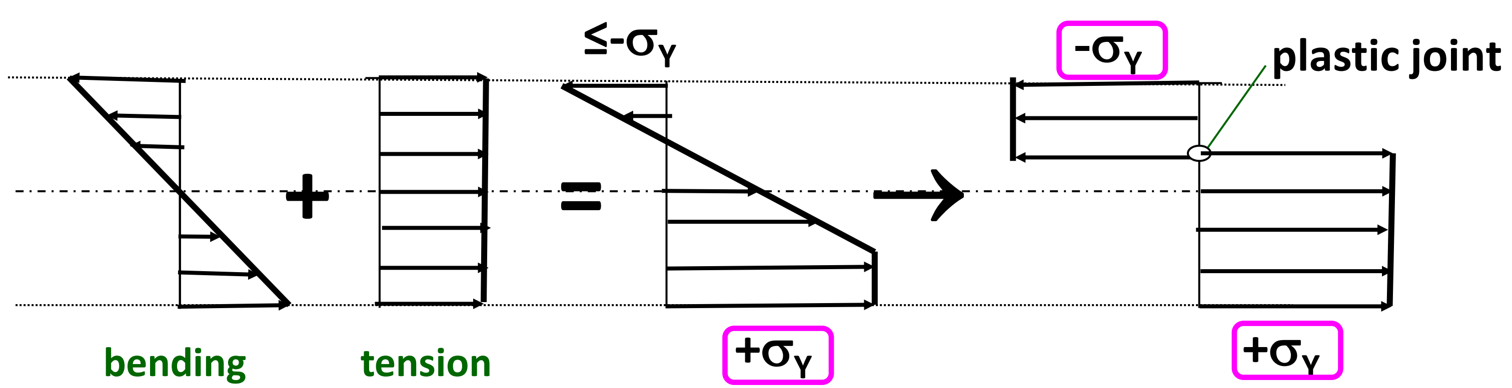


Thirdly we suppose that the beam is loaded with the axial force F_P and moment M_P

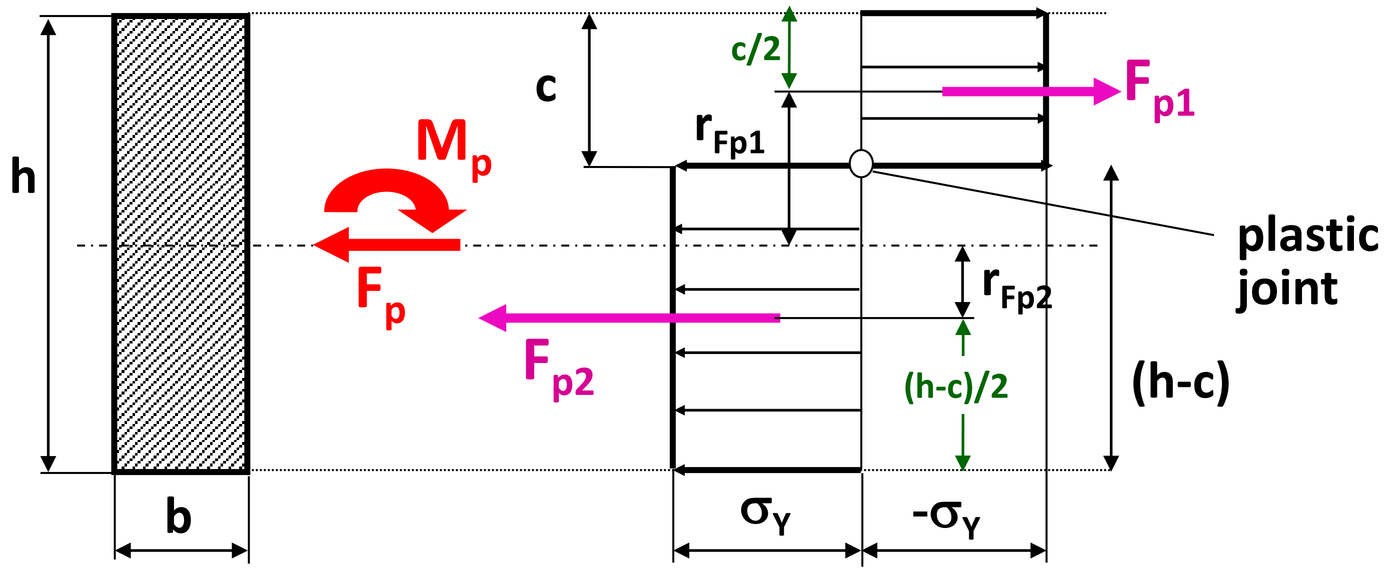
Superposition of both loading in elastic state



Superposition of both loading in partially plastic and finally in fully plastic state



Superposition of both loading in fully plastic state



F_p = max. force for fully plasticized profile; M_p = max. moment for f.p.p.

In a distance c from surface a plastic joint arises.

We specify limiting forces and their moments calculated to the profile axis and from them total limiting values:

$F_{P1} = c * b * \sigma_Y$ $\qquad$ $F_{P2} = (h - c) * b * \sigma_Y$

$$F_P = F_{P2} - F_{P1} = \underbrace{(h - c) * b * \sigma_Y}_{} - \underbrace{c * b * \sigma_Y}_{} = (h - 2c) * b * \sigma_Y$$

Moment arms of forces $F_{P1,2}$ and the resulting moment

$r_{FP1} = h/2 - c/2 = (h - c) / 2$ $\qquad$ $r_{FP2} = h/2 - (h-c)/2 = c/2$

$$M_P = F_{P1} * r_{FP1} + F_{P2} * r_{FP2} =$$

$$M_P = c * b * \sigma_Y * (h - c) / 2 + (h - c) * b * \sigma_Y * c/2 = c * (h - c) * b * \sigma_Y$$

Ratio of limiting values for separate and combined loading is after modification:

loaded with only $F_{P0} = h*b*\sigma_Y$ when σ_Y is reached in all strings

loaded with F_P and M_p

loaded with only $M_{P0} = ¼*b*h^2*\sigma_Y$ when the profile is fully plasticized

$$F_P / F_{PO} = 1 - 2c/h \qquad M_P / M_{PO} = 4\, c/h * (1 - c/h)$$

loaded with F_P and M_p

$c/h = (1-F_P/F_{PO})/2$

$M_P/M_{PO} = 4/2*(1-F_P/F_{PO})*(1-1/2*(1-F_P/F_{PO}))$

If we eliminate the unknown ratio c/h from these equations we obtain this equation:

$$M_P / M_{PO} + (F_P / F_{PO})^2 = 1$$

Limiting loads for elastic and fully plasticized profiles are:

for superposition $\boxed{M_P = \sigma_M * W}$ (moment + force) $\boxed{F_P = \sigma_F * S}$

for plastic state $M_{P0} = 1.5 * \sigma_Y * W$ (only moment) $F_{P0} = \sigma_Y * S$ (only force)

and after substitution of these values in the previous equation is

where are: σ_F = limiting stress for loading with force

σ_M = limiting stress for loading with bending moment

W cross-section modulus

S profile cross-section

$$\frac{\sigma_M}{1.5 * \sigma_Y} + \left(\frac{\sigma_F}{\sigma_Y}\right)^2 = 1$$

(for superposition of both loadings)

If we introduce a total stress $\sigma_\Sigma = \sigma_M + \sigma_F \rightarrow \sigma_M = \sigma_\Sigma - \sigma_F$ we obtain this very important equation

$$\boxed{\frac{\sigma_\Sigma}{\sigma_Y} = 1.5 + \frac{\sigma_F}{\sigma_Y} - 1.5 * \left(\frac{\sigma_F}{\sigma_Y}\right)^2}$$

What follows from the equation for various beam loading?

Only bending moment (no axial force)

$$\frac{\sigma_\Sigma}{\sigma_Y} = 1.5 + \cancel{\frac{\sigma_F}{\sigma_Y}} - 1.5 * \left(\cancel{\frac{\sigma_F}{\sigma_Y}}\right)^2$$

$F = 0 \rightarrow \sigma_F = 0 \rightarrow \sigma_F / \sigma_Y = 0$ Than is

$$\sigma_\Sigma / \sigma_Y = 1.5$$

$\rightarrow$ see the previous result

„Total stress σ_Σ“ in the beam can be 1.5 times higher than σ_Y.

Only tensile (or compression) load (= only axial force, no bending)

$M = 0 \rightarrow \sigma_M = 0 \rightarrow \sigma_{Fmax}$ can be $= \sigma_Y \rightarrow \sigma_F / \sigma_Y = 1$ Than is

$\sigma_M = \sigma_\Sigma - \sigma_F$

$$\sigma_\Sigma / \sigma_Y = 1.5 + 1 - 1.5*1^2 = 1.0$$

$$\frac{\sigma_\Sigma}{\sigma_Y} = 1.5 + \frac{\sigma_F}{\sigma_Y} - 1.5 * \left(\frac{\sigma_F}{\sigma_Y}\right)^2$$

Total stress σ_Σ in the beam can be maximal equal to σ_Y.

σ_F for tension (compression) cannot be higher than σ_Y (for elast. plast. model)

Combination of bending moment and tension (compression)

For ratio $\sigma_F / \sigma_Y = 0.333$ $\rightarrow$ Than is

(For ex. the beam is loaded from 33.3 % with a tension and from 66.6 % with a bending moment)

$$\sigma_\Sigma / \sigma_Y = 1.5 + 0.333 - 1.5*0.333^2 \approx 1.67$$

Total stress σ_Σ in the beam can be 1.67 times higher than σ_Y.

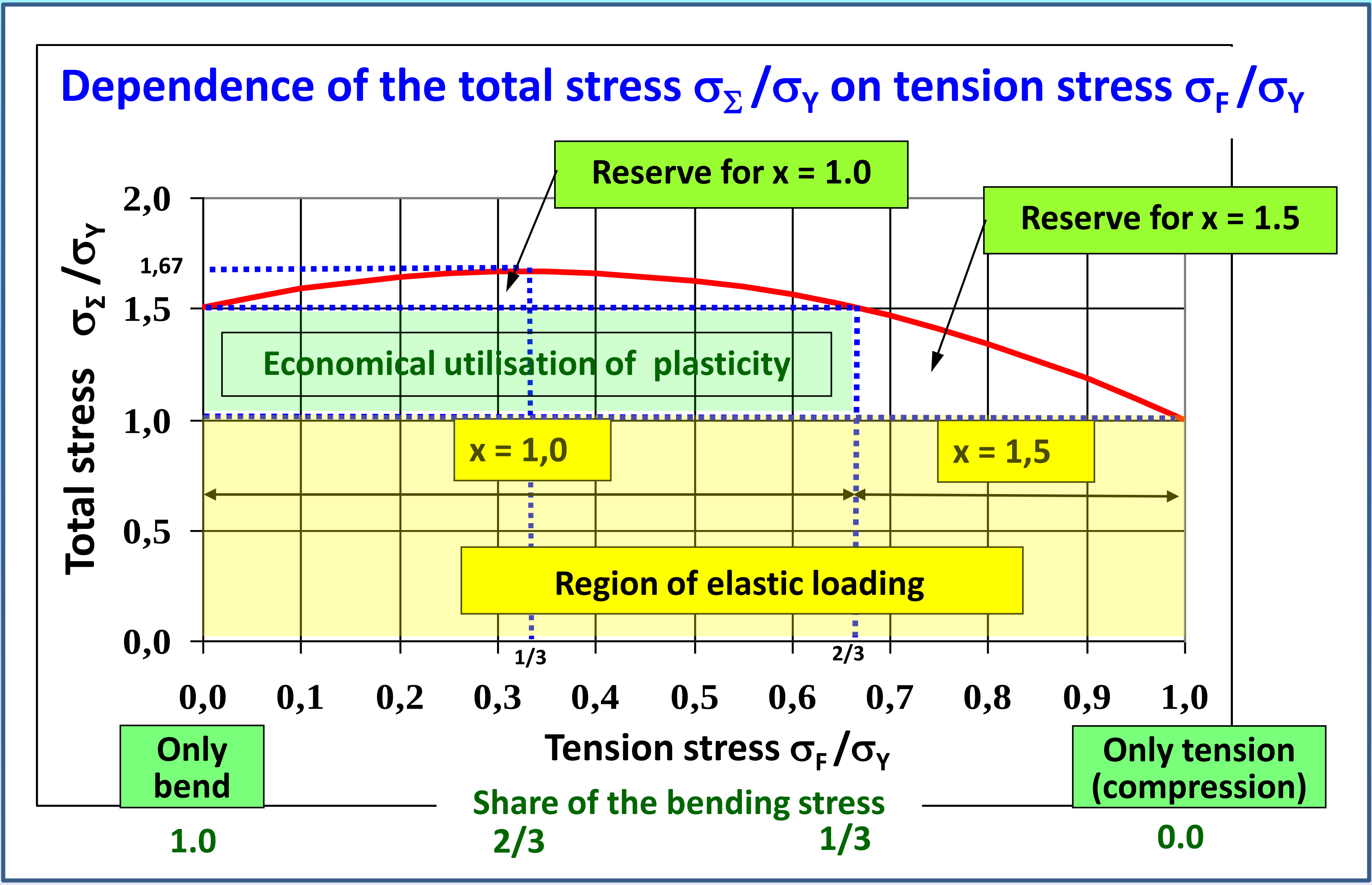
For ratio $\sigma_F / \sigma_Y = 0.666$ $\rightarrow$ Than is

(For ex. the beam is loaded from 66.6 % with a tension and from 33.3 % with a bending moment)

$$\sigma_\Sigma / \sigma_Y = 1.5 + 0.666 - 1.5*0.666^2 \approx 1.50$$

Total stress σ_Σ in the beam can be 1.50 times higher than σ_Y.

Dependence of σ_Σ / σ_Y on σ_F / σ_Y we can set in the following diagram



When is the proportion of the total bending load of a beam in the range of 33 - 100%, we can use the safety factor x = 1.0

- **From this dependence follows that <u>for only bending loading is total limiting stress equal 1.5 x yield stress.</u>**

- **For combined loading (tensioning + bending) with prevailing bending stress the total limiting stress (acceptable) rises up and the maximum is for the ratio $\sigma_F / \sigma_Y = 1/3$, when is the total limiting stress $\sigma_\Sigma = 1.67 * \sigma_Y$.**

- **<u>Than it falls down and for only pure tensioning it reaches value $\sigma_\Sigma = \sigma_Y$</u>.**

<u>This finding we can use for the choice of the safety factor</u>.

These results can be used only for a bending moment or its combination with a tension (compression). However, for torsion this method can not be used (shear stress)!!!

- **A structure in what are only tensioning or compression stresses collapses (according the elastic limit state) when in some part (parts) the yield point is reached, it is when $\sigma_F = \sigma_Y$. For membrane stress (= only tension or compression) the safety factor x = 1.5 ($\sigma_{AF} = \sigma_Y / x = \sigma_Y / 1.5$) is used.** **($\sigma_{AF}$ = allowable stress for tension)**

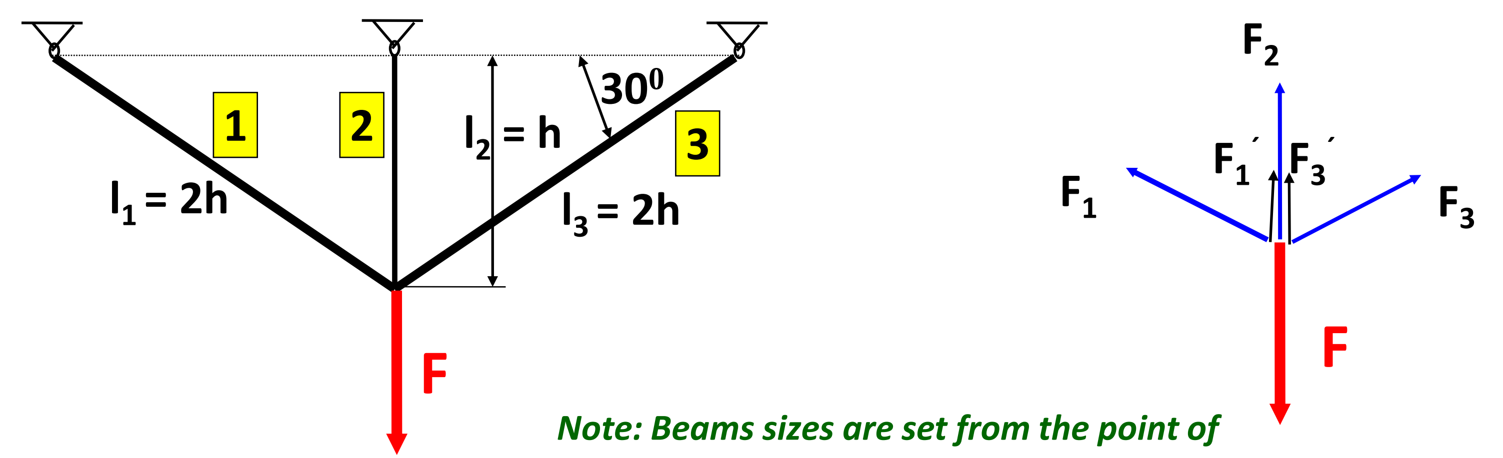
- **If in a structure is only pure bending we can use safety factor x = 1.0 (we have reserve 50 % to limiting state - $\sigma_{AM} = \sigma_Y / x = \sigma_Y$).**

  (σ_{AM} = allowable stress for bending moment)

- **For the combination of tensioning and bending is this reserve higher compared with only bending. Therefore is for the case till the ratio $\sigma_F / \sigma_Y = 0.67$ possible to use the safety factor x = 1. For higher ratio values we can use the safety factor value again x = 1.5.** **($\sigma_A = \sigma_D$ according Czech practice)**

5. Plasticizing of statically indeterminate structures with only uni-axial tensioning

As example we have a structure made from 3 beams that is on this fig. The structure is loaded with force F.



Note: Beams sizes are set from the point of view of illustrative results (integers).

The structure has following parameters:

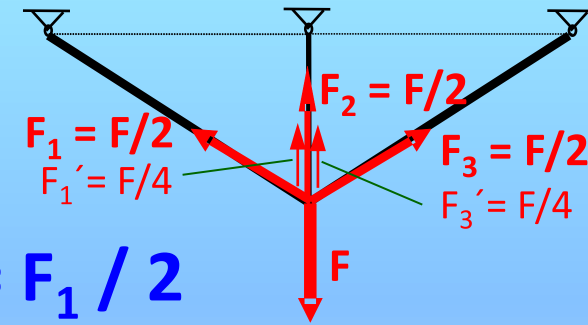
Beams lengths: $l_2 = h$ $l_1 = l_3 = 2*h$

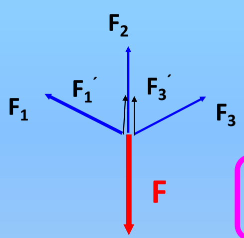
Beams sections: $A_2 = A$ $A_1 = A_3 = 2*A$

a) The structure is loaded with force F that is in range $F_{el} < 0; 2*A*\sigma_Y >$.

The middle beam 2 will behave during the loading and unloading elastic (its deformation will be only elastic).

Balance of power in the structure is:





axial components of forces F_1 and F_2

$$F_1 = F_3 \qquad F_1{}' = F_3{}' = F_1 * \sin 30° = F_1 / 2$$

$$\boxed{F = F_1{}' + F_2 + F_3{}'} = F_1 / 2 + F_2 + F_1 / 2 = F_1 + F_2$$

balance of forces in the vertical direction

For loading with force $F = 2*A*\sigma_Y$ in the central beam 2 will be stress (providing that $F_2 = F_1 = F_3 = F/2$ – it is specified from requirement that all beams have to have the same extension in the F direction and from the beams stiffness):

$$\boxed{F_2} = F / 2 = \underbrace{2*A*\sigma_Y}_{F_{emax}} / 2 = A*\sigma_Y \rightarrow \sigma_2 = F_2 / A = A * \sigma_Y / A = \boxed{\sigma_Y}$$

From it follows that till the force is reached the beam 2 (with cross section = A) has only elastic deformations.

In beams 1 and 3 is the same force, but they have a cross-section 2A $\rightarrow \sigma_{1,3} \leq \sigma_Y / 2$

b) Now we suppose that the structure is loaded with higher force that will be in the range $F_{pl} <2*A*\sigma_Y; 4*A*\sigma_Y>$.

- Under these conditions the beam 2 starts to deform plastically, but beams 1 and 3 have still the elastic deformation.
- After unloading beams 1 and 3 want to go to their original position but the beam 2 is elongated. Therefore in the beam 2 arise after the unloading compression pre-stress and in beams 1 and 3 tension pre-stress.
- For a new loading with force increasing from 0 to F_{pl} the structure will behave like elastic system in all this range (see the next fig.).

(2 is the most loaded beam)

- For the new loading in the central beam 2 will be following stresses: - compression stress (pre-stress) $\rightarrow$ no stress $\rightarrow$ tension stress. (remember the beam after its plasticizing $\rightarrow$ the structure is adapted to the 1^{st} overloading)

c) For loading of the structure with even higher force F that will be in range (F_{II} <$4*A*\sigma_Y$; $5*A*\sigma_Y$>)

$F_1 = F_3 = F/2 = 4*A*\sigma_Y/2 = 2*A*\sigma_Y \rightarrow$

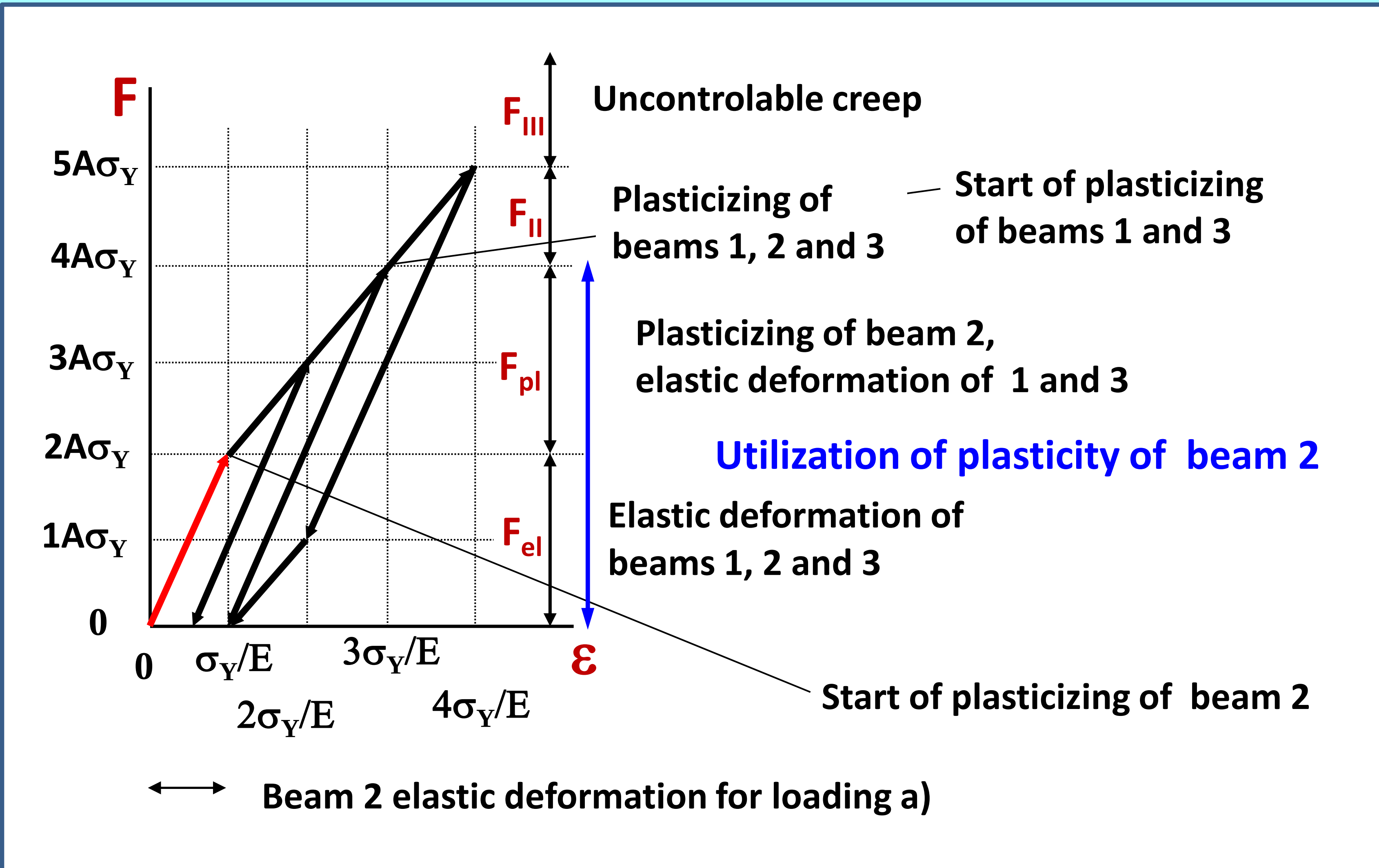
$\sigma_1 = \sigma_3 = 2*A*\sigma_Y/2*A = \sigma_Y$

- Now plastic deformations in beams 1 and 3 rise too.
- After unloading of the structure in the beam 2 arises pressure plastic deformation (alternating plasticizing tension + compression $\rightarrow$ danger of the fatigue failure of the beam 2).

d) For loading of the structure with the force higher than $F_{III} > 5*A*\sigma_Y$

- Plastic deformations are in all beams and the structure comes to a region of uncontrollable creeping.
- For hardening materials it is possible to load such structure but in praxis is not the hardening calculated (higher safety).

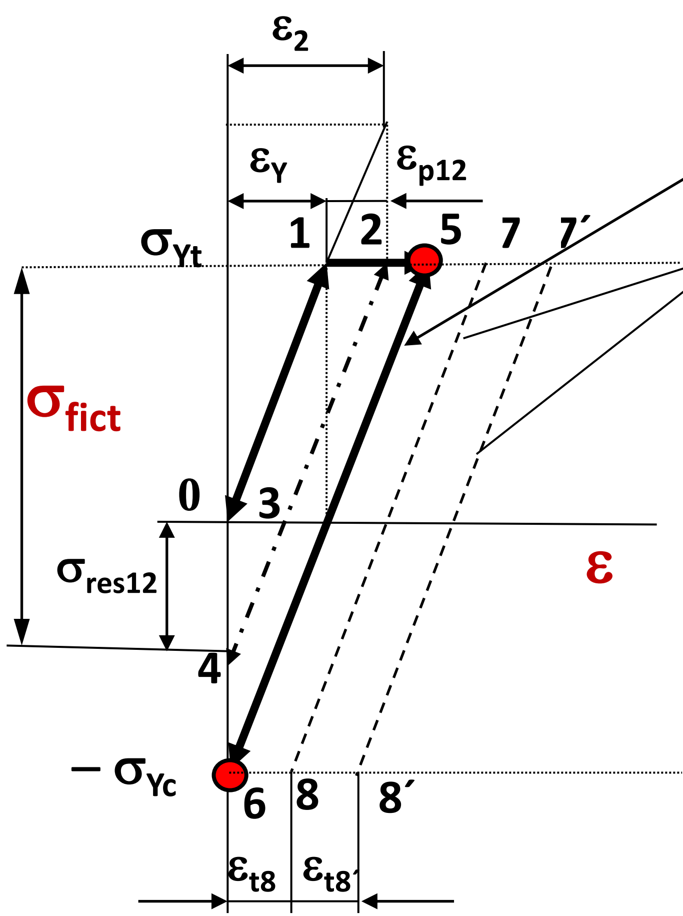
$\rightarrow$ danger of fatigue failure of the beams 1 and 3 too



Loading of 3 beams structure according previous fig. (p.27)
Situation of the middle beam 2.

(more illustrative is the following Planck diagram)

Now we will consider an ideally elastic-plastic material. It means that the beam 2, when it reaches the yield point, starts to deform plastically (on line σ_Y = const.). The course of the beam 2 alternating loading and unloading is shown in so-called Planck diagram (fig.).



Max. possible adaptation of the structure to the overloading with force F

Alternating plasticizing of beam 2

0-1-0	elastic deformation of beam 2
0-1-2	overloading of beam 2 and its plastic deformation (but elastic of 1 and 3)
2-3-4	structure unloading and compression residual stress in beam 2 = its prestress
0-1-2-5	max. possible loading of the structure
5-6	after the structure unloading is in the beam 2 residual stress – σ_{Yc} = its max. prestress
6-5	during new loading the structure withstands 2 x > loading → structure adapted on this overloading
0-1-7-8 ...	during next repeating overloading and unloading alternate plasticization +/- comes → fatigue failure of the structure

In the region of elastic deformation the state of beam 2 moves on the line 0 – 1. When the yield point is reached the beam starts plastically deform on the line 1 – 2.

After unloading (line 2 – 4) a compression pre-stress arises (residual stress) in the beam.

$$\sigma_{res12} = - E * \varepsilon_{p12}$$

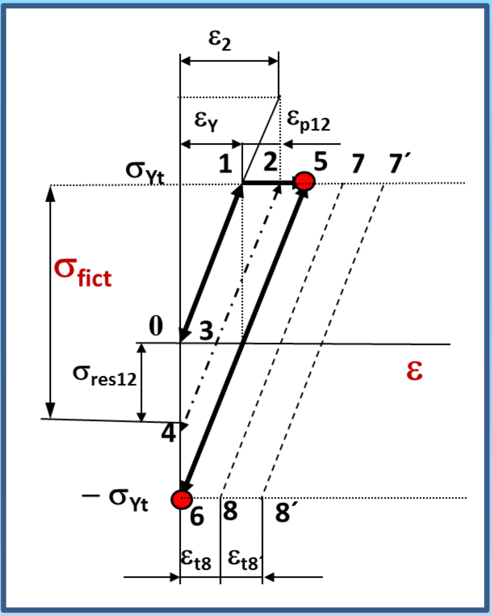
If the structure is loaded again so that $\varepsilon = \varepsilon_2 = \varepsilon_Y + \varepsilon_{p12} \leq 2/E*\sigma_Y$ the repeated loading will go in the elastic state on the line 4 – 2. In so doing it does not depend on this how many times was the point 2 reached (independent on a loading history).

In the case the structure is adapted to this overloading.

For a new reach of the yield point in the beam 2 (point 2) a following stress is necessary (line 4 – 2):

$$\sigma = \sigma_Y + \sigma_{res12} = \sigma_{fictive}$$

- When is the structure loading so high that plastic deformation reaches the value $\varepsilon_p = \varepsilon_Y$ (total deformation is $\varepsilon = \varepsilon_5 = 2^* \varepsilon_Y$; it is the point 5), reaches the value of the residual stress after unloading $\sigma_{res12} = - \sigma_{Yc}$ (point 6).
- In the case the beam 2 can withstand the stress $\sigma_{fictivemax} = 2^*\sigma_Y$ = the structure maximal adaptation on its overloading.



- For a next load rising and resultant next rising plastic deformation (point 7) the beam 2 starts to plastic deform after unloading in the compression region too (point 8). For the next overloading and unloading plastic deformations in tensile and compressive regions arise (points 7´and 8´).

Next alternating overloading and unloading (on lines 6 – 5 – 7 – 8 - 7 – 7´ - 8´ ...) faces to the alternating plasticizing of the beam profile with result of fatigue failure (with always increasing permanent deformation).

The structure is not able to adapt itself to such repeated overloading.

→ fatigue failure

→ for this statically indeterminate structure is after its overloading the maximal load in the elastic region

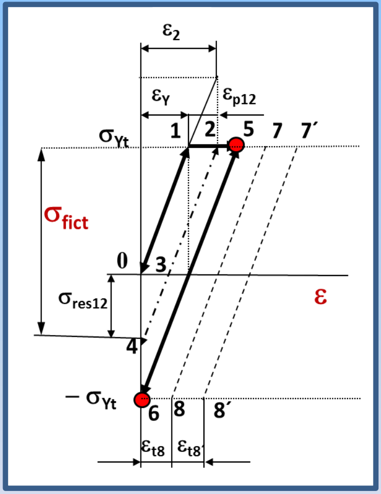
$$\boxed{\sigma_{fict.limit.} = 2 * \sigma_Y}$$

The structure adaptation against an overloading can be only in the region of deformations $\langle \varepsilon_Y; 2\varepsilon_Y \rangle$.

About possibilities of the adaptation a dimensionless parameter called the coefficient of adaptation k_p decides (shake down, Einspieltheorem).

It is done by the ratio of loading in the second cycle F_2, when the structure starts to plasticize in tension and compression regions to loading F_1 when the structure reaches for the first the yield point in tension:

$$k_p = \frac{F_2}{F_1} = \frac{\overline{65}}{\overline{01}} \leq 2$$



***Note:** These relations are valid only for uni-axial stress. E.g. for the tri-axial stress are these relations much more complicated, but the principle and results are similar.*

- **Loading of statically indeterminate structures with sudden shape changes or other discontinuities causes a rise of local secondary and peak tensions (stresses) in these places.**

- **These tensions are in the wall section distributed unevenly. Stress peaks have only local character.**

- **So that from it following plastic deformations do not expand into the peak surroundings. From it follows that a high local overrun of the yield point has not effect on the total strength of the structure (system, pressure vessel etc.).**

That is why for a static loading stress peaks are not taken into account!

(it is valid only for a vessel wall thickness specification, but they can have effect on the fatique loading)

For steel shells loaded mainly with membrane stress following **reach of these local (transitional) stresses L is specified:**

$$L = \sqrt[4]{\frac{R^2 * s^2}{3*(1-\mu^2)}} \approx 0.78*\sqrt{R*s} = 0.55*\sqrt{D*s}$$

theoretical value of stress peak reach

For technical praxis a value with safety factor 3 is assumed

$$L_K = 1.65*\sqrt{D*s}$$

stress peak reach

where R = D/2 is a radius of curvature in a calculated place (D is a diameter) and s is a wall thickness of the membrane (shell). μ is Poisson constant and for steel it is μ ≈ 0.3.

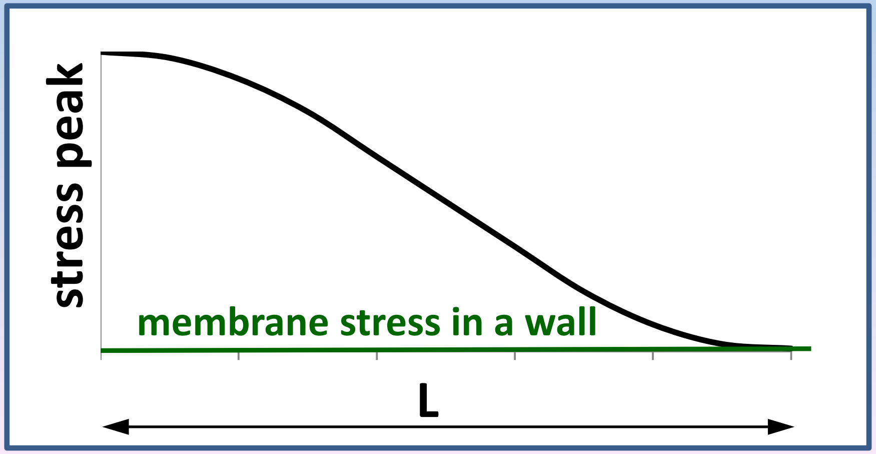
Ex.: *D = 2000 mm; s = 10 mm; L = 0.78* √(1000*10) = 78 mm;*
L_K = 1.65 √(2000*10) = 233 mm*

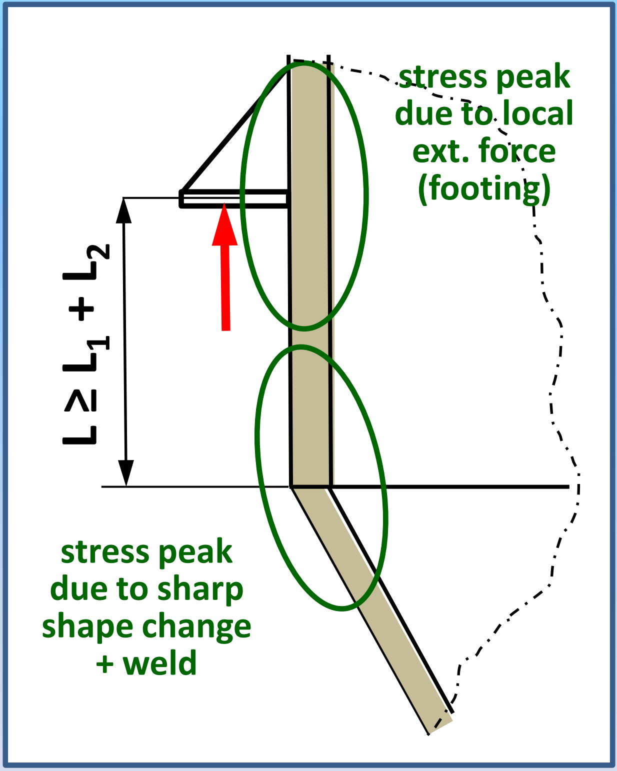
Adjoining stress raisers (stress peaks) = local external forces, welds, notches, sharp shape changes etc. **must have a distance**

$$L \geq L_1 + L_2$$

or with the 300 % reserve

$$L_K \geq L_{K1} + L_{K2}$$





Stress peaks would not be cumulated! (would not overlap)

6. Stress categories in a structure (membrane wall)

(Types of stresses that can be in a wall)

(we can specify them from the external and internal forces balance → $F_{ext} = F_{int}$)

1. **Primary stress** = tensile and compressive stresses (membrane stresses) in a wall that are distributed uniformly or bending stresses that are caused by external forces.

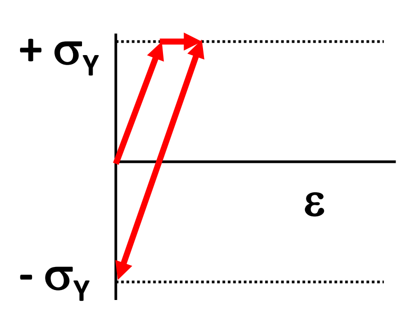
- After arising of a plastic deformation in some strings (parts) of a profile (when in the place the yield point is exceeded) they do not decrease too much. (remember the bar loaded with axial force)
- That is why these primary stresses tensile and compressive are limited with the safety factor x = 1.5 (exceeding of the loading owing to working conditions or incorrect dimensioning can cause a rise of a big plastic deformation with following failure).
- Bending stresses and combined stresses (e.g. bending + tensile) can have safety factor x = 1. In the case the value of an allowable stress must not exceed the yield point (theoretically for elastic-plastic mater.).

2. Secondary stress = statically indeterminate stresses that after exceeding the yield point do not cause bigger plastic deformations and after a some plasticization are able to adapt to an local overloading (effect of the static indeterminacy).

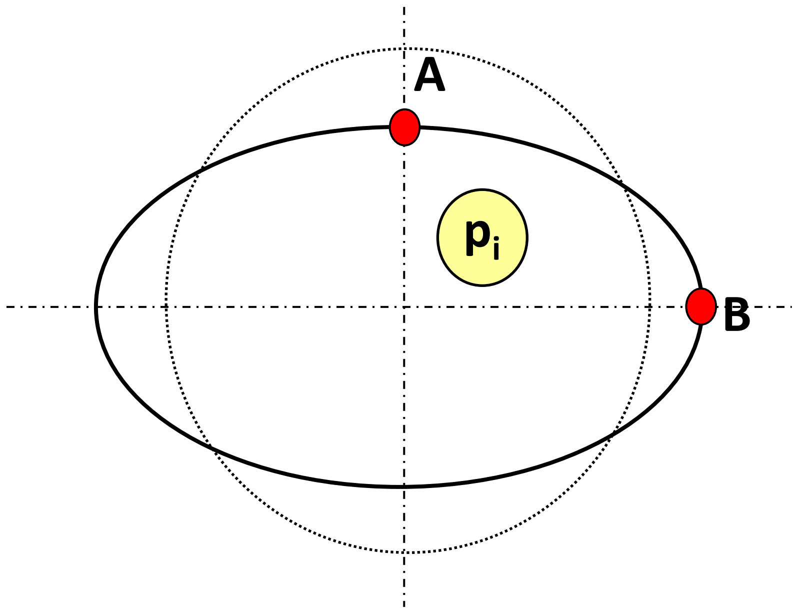
- In this group belong for example stresses caused by local external forces, membrane shape change, temperature change, high heat flux in wall etc.

- A value of these secondary stresses themselves or with combination (sum) with primary stresses is limited by the requirement that their maximal value in any direction and any string has to be lower than is double the yield point.

(remember the Planck´s diagram)

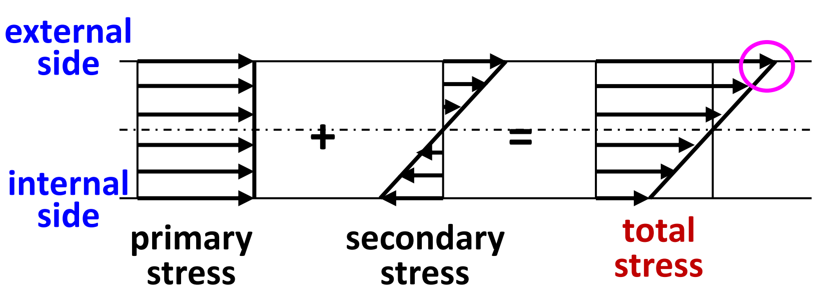


Example of the secondary stress in a shell (elliptical tube) with internal overpressure p

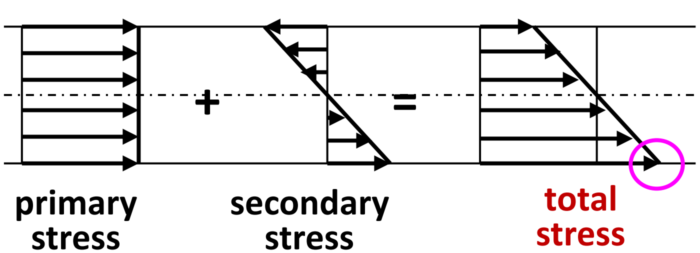


Owing to the inter. pressure the elliptical tube shape wants to change into the circular ones →
secondary bending stress in wall
(primary stress = tension)

Place A tends to the lower radius
→ outer strings have tension, inner compress.



Place B tends to the bigger radius
→ outer strings have comp., inner tension



$\sigma_{ps} = p_i*D/2s$

plasticization can be used here (stress peaks)

Similar situation is in a wall with a high heat flux – see part 13

- For a proper design we can expect, in places with the highest stress, partial plasticization and residual stresses after unloading (= adaptation on this local overloading). → they must be < σ_Y !!
- But if these residual stresses (pre-stresses) reach the yield point there is a danger of an alternating plasticization that causes the fatigue failure (with big contractions) → low-cycle fatigue.
- The adaptation of a structure or pressure vessel to an overloading is used for pressure tests (elimination of possible stress peaks). A distance on what these secondary stresses act depends on a vessel diameter and its thickness *(L = 1.65*√(D*s)*.

These stress peaks are again eliminated during the vessel pressure test.

3. Stress peaks (tertiary stresses) = stresses with local character that are only in some profile strings (local external forces, sharp deviations of form, welds, notches, material changes etc.

Local plastic deformations practically have not effect on a structure (shell ...). *(only in this distance L = 1.65*√(D*s)*

They are taken into account only for low-cycle fatigue (e.g. number of cycles of a structure loading and unloading).

7. Conditions of plasticity for stress in more axis

In the case we have to specify an equivalent stress σ_e, that will be compared with the yield point σ_Y. The equivalent stress is given by 6 components of a stress tensor (σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{xz}).

stress in axis x, y, z
shear stress in planes

$\rightarrow$ we must re-count the real stress in a structure to 1 value that is compared with the allowed stress (as the σ_Y was specified during a uniaxial tear test)

Conditions of plasticity determined by various theories differ in a way of a specification of the equivalent stress. As the plastic deformation takes place usually by shear there are used various forms of shear stresses for the determination of the equivalent stress.

In engineering praxis following theories for biaxial or triaxial stress are used (in next text we will suppose that $\sigma_1 > \sigma_2 > \sigma_3$). The equivalent stress determined according these theories is compared with the yield point ($\sigma_e < \sigma_Y$) or with allowed stress σ_D.

1. Hypothesis of maximal normal stress σ_{max} (tensile or compressive stress)

Lameé for: 2 axial stress $\sigma_e = \sigma_1$ → σ_e = the max. stress in any axis

3 axial stress $\sigma_e = \sigma_1$ (tension or pressure)

(depending on $\sigma_{Ytension}$ and $\sigma_{Ypressure}$)

→ According this hypothesis a structure failure happens when a maximal stress in some axis reaches a value equal to a limiting stress = yield point

2. Hypothesis of maximal shear stress

Guest: 2 and 3 axial stresses

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} \leq \tau_Y = \frac{\sigma_Y}{2}$$

According this hypothesis a structure failure comes when a maximal shear stress τ_{max} in any plane reaches value equal to a limiting shear stress for uniaxial tensile τ_Y. The condition is expressed by the following relation $\sigma_e = \sigma_1 - \sigma_3 \leq \sigma_Y$.

In the case of biaxial stress is $\sigma_3 = 0$.

Than a region of safe loading is given by following conditions:

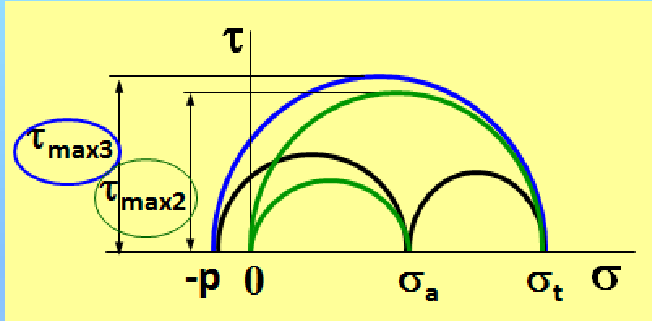
$$|\sigma_1 - \sigma_2| \le \sigma_Y \qquad |\sigma_1| \le \sigma_Y \qquad |\sigma_2| \le \sigma_Y$$

$\boxed{\tau_{max2} = \sigma_1 / 2}$

or for triaxial stress

$$|\sigma_1 - \sigma_3| \le \sigma_Y \qquad |\sigma_1| \le \sigma_Y \qquad |\sigma_3| \le \sigma_Y$$

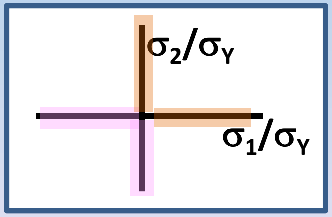
$\boxed{\tau_{max3} = (\sigma_1 + \sigma_3) / 2}$



Mohr´s circles for our example

For satisfying of these conditions of a structure (shell ..) a maximal value of a resulting stress must lie inside so-called Tresc´s hexagon (see fig.later).

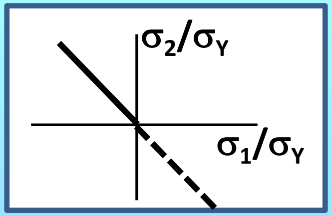
Now I show you some examples of various loadings and them equivalent stresses in the Tresc´s hexagon.



2a) Uni-axial tension in the axis σ_1/σ_Y direction; $\sigma_1 \le \sigma_Y$; $\sigma_2 = 0$
in the axis σ_2/σ_Y direction; $\sigma_2 \le \sigma_Y$; $\sigma_1 = 0$

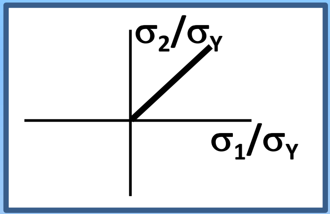
2b) Uni-axial compression in direction of axis $-\sigma_1/\sigma_Y$; $-\sigma_1 \le -\sigma_Y$; $\sigma_2 = 0$
in direction of axis $-\sigma_2/\sigma_Y$; $-\sigma_2 \le -\sigma_Y$; $\sigma_1 = 0$

2c) Simple shear $|\sigma_1| = |\sigma_2|$; $\sigma_1 = -\sigma_2$



2d) Thin-walled spherical shell with inner overpressure p_i

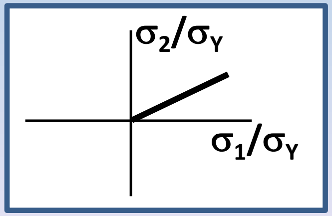
Tangential (circumferential) stress = axial stress → stress in all directions is the same



$$\sigma_a = \sigma_1 = \sigma_t = \sigma_2 = p_i * r / 2s$$

Similarly it is for outer overpressure (but for the case we have to take into account not only the stress but conditions of stability too – see later in part 8)

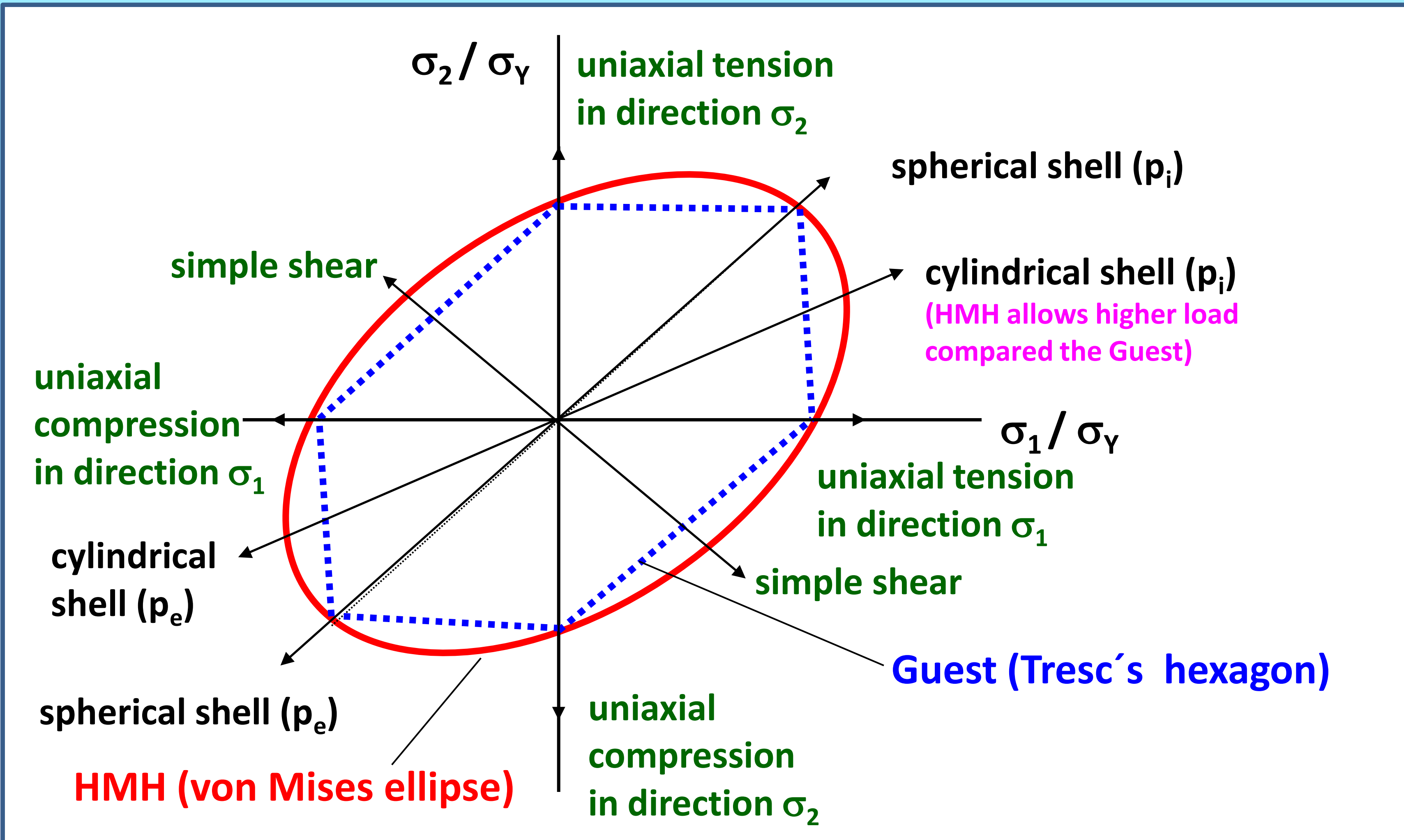
2e) Thin-walled cylindrical shell with inner overpressure p_i



Tangential stress = 2 * axial stress *(see later)*

$$\sigma_a = \sigma_2 = p_i * r / 2s; \qquad \sigma_t = \sigma_1 = p_i * r / s = 2*\sigma_2$$

Conditions of plasticity for uni- and biaxial stresses



Inside is a region of a safe loading

3. Hypothesis HMH
(energetic hypothesis Huber – Mises – Hencky)

According this hypothesis the biggest effect on a structure failure has a specific energy of stress needed for a shape change (elongation, compression, contraction ...). For the case of a plane stress (bi-axial) it is valid that:

$\sigma_e^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1 * \sigma_2$ or $\sigma_e = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 * \sigma_2}$

If we divide both sides of the equation by σ_Y and bring these results in the previous fig. we obtain an ellipse that goes through vertices of the Tresc´s hexagon.

$$\boxed{\sigma_{eGuest} \leq \sigma_{eHMH}}$$

Comparing these two hypothesis we can see that the Guest hypothesis lies on the side of higher safety. The HMH hypothesis is more economical (it better utilizes a material strength – for simple shear and cylindrical shells; not for simple tension / press and spherical shell).

In the case of tri-axial stress is a range of a safe stress a "hexagonal prism with pyramids" for the Guest´s hypothesis, in the case of HMH hypothesis it is a spheroid (circular ellipsoid).

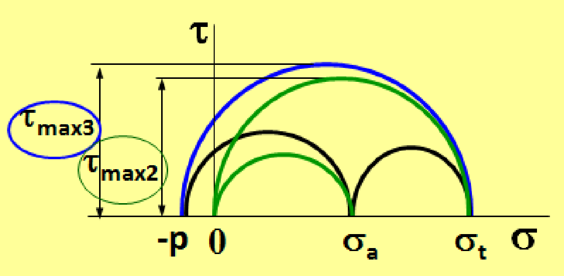
Example:

Let us suppose a thin-walled cylindrical shell with inside radius r = 500 mm and wall thickness s = 20 mm, that is loaded with an internal overpressure p_i. The yield point of the shell material is σ_Y = 230 MPa. Safety factor x = 1.5.

Our task is to specify maximal allowed overpressure p_D.

$k = d_e/d_i = 2*(500+20) / (2*500) = 1.04 < 1.1$ OK = thin-walled

It is valid that (see later)



tangential stress: $\sigma_t = p*r/s$ axial stress: $\sigma_a = p*r/2s$

radial stress: $\sigma_r = -p$ and $\sigma_t > \sigma_a > \sigma_r$

- According the condition σ_{max} (Lameé hyp.) is $\sigma_e = \sigma_t$
- According the bi-axial Guest condition is σ_e the same $\sigma_e = \sigma_t$ $(\tau_{max} = \sigma_t / 2 \leq \tau_Y / x = \sigma_Y / (2*x)$ $\rightarrow$ $\sigma_e = \sigma_t \leq \sigma_Y / x)$

$\sigma_t = p * r / s \leq \sigma_Y / x$ $\rightarrow$

For 3D $\sigma_e = \sigma_t + p \leq \sigma_D$
$p = \sigma_D / (r/s+1) = ... = 5.90$ MPa

$p_D = \sigma_Y * s / (x * r) = 230*20/(1.5*500) = 6.13$ MPa

- **According the energetic condition HMH is**

$$\sigma_e = \frac{1}{\sqrt{2}} * \sqrt{(\sigma_t - \sigma_a)^2 + (\sigma_a - \sigma_r)^2 + (\sigma_r - \sigma_t)^2}$$

For easier comparison of our results with the Guest hyp. we introduce symbols

$a = \sigma_a / \sigma_t = 0.5$;

$b = \sigma_r / \sigma_t = p / (p*r/s) = -230*20/(230*500) = -0.04$

and after substitution and modification we obtain

$$\sigma_e = \frac{\sigma_t}{\sqrt{2}} * \sqrt{(1-a)^2 + (a-b)^2 + (-b-1)^2} = ... = 0.901 * \sigma_t$$

$\approx 0.90*\sigma_{e2D\ Guest}$

$\approx 0.86*\sigma_{e3D\ Guest}$

$$\sigma_e = 0.901 * \frac{p*r}{s} \le \frac{\sigma_Y}{x}$$

where $x = 1.5$

Than the maximal inner overpressure calculated from the hypothesis (for plasticity conditions) is

$$p_p = \frac{\sigma_Y * s}{0.901 * r} = \frac{230 * 20}{0.901 * 500} = 10.21.MPa$$

and allowed overpressure is $p_D = p_p / x = 10.21 / 1,5 = 6.81$ MPa

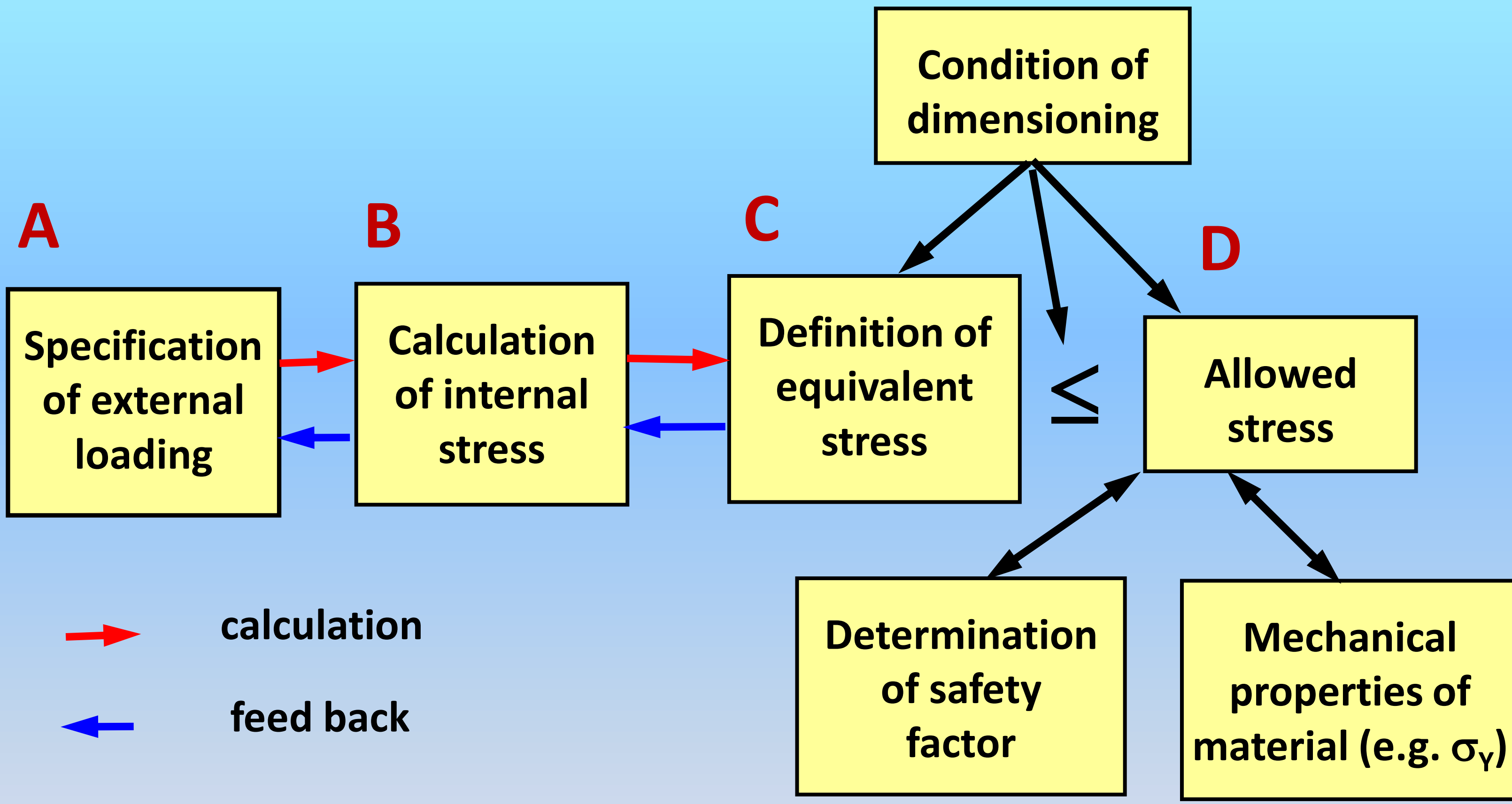
(x according Guest 2D it was 6.13 MPa and Guest 3D it was 5.90 MPa).

8. Basic methods of equipment dimensioning (vessels, parts etc.)

Data necessary for the equipment dimensioning:

- **Specification of processes that are in an equipment**
  (working parameters and their change in time = temperatures, pressures, heat flux → additional thermal stress, effect of thermal dilatations, fatigue loading ...)
- **Mass and energy inlets and outlets = balances**
  (mass of treated material → external forces acting on the structure, inertial forces ...)
- **Control system (regulation, sensors, actuating devices, way of control, parameters fluctuation etc.)**
  (its accuracy and sensitivity → working parameters variations ...)
- **Location in a line (= effects of surroundings, footings, lifting necks or eyes, connecting flanges and pipes etc.).**
  (e.g. forces caused by piping thermal dilatations, mass of connected pipes ...)

Chart of a way of an equipment dimensioning



e.g. pressure, mass of treated material, mass of equipment, single forces & moments, thermal dilatations ....

e.g. from a balance of external and internal forces (= stresses)

according a proper hypothesis

from a data sheet of a chosen material or a material certificate

A) Specification of external loading

During this specification a designer has to solve following problems:

- Possibility of separation of a combined loading in simple loadings and subsequent utilization of laws of superposition of stresses and shifts (elongations etc.) = a problem simplification

- Character of external forces (surface forces = out of material = internal or external pressure, local forces, moments, treated material mass ...; volumetric forces = in material = mass of equipment, internal stresses owing to welding or fabrication, thermal stresses owing to dilatation ...)

- What is an effect of the loading = monotonous or changing (periodical or casual); way of operation (continuous or discontinuous).
  $\rightarrow$ effect of the low cycle fatique

Continuous operation	- at approximately constant parameters	good working cond.
	- at variable parameters	
Discontinuous operation	- at approximately constant parameters	
	- at variable parameters	the worst working conditions

- **Conditions of a loading character and transient states (way of operation and its limiting conditions, way of starting and breakdown (after every shift or in longer intervals, fast or slow achievement of working parameters), equipment maintaining etc.).**

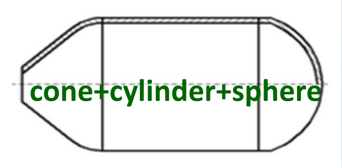
- **Further operational conditions or restrictions (operation with lower or higher output (e.g. catching up a loss after a failure of production), allowed variation of operating temperatures and pressures, heating or cooling speeds and number of their changes, corrosive and abrasive effects of working media, fouling on working surfaces during operation, ways of cleaning and sanitation etc).**

- **$\rightarrow$ these working conditions have effect on the low cycle fatigue**

B) Internal stress specification

For a specification of the internal stress it is necessary to solve following problems:

- Is it possible to replace a component (= a part of an equipment) with a simple geometric form (sphere, cylinder, plate, beam, lattice structure, frame etc.) or their combination?
  (a stress in such simple geometric form we can calculate easy)
- Is it possible to analyze the component or equipment as single elements for what we know solution and calculation of internal forces (stresses) and moments and we are able to determine boundary conditions for their connection?
  (→ easier calculation or we can use a computer program)



- Is it possible or have we to use a given method of calculation according standards, rules etc.?
  (pressure vessels, supporting structures ... have to be designed according standards)

→ effort to simplify the solution

- **Unless, is it possible to use some simplifying presumptions** and so the problem approximately solve? What is an error of such approximate solution? (simplification of shape, load, neglecting of some parameter...)

- **Is the structure etc. statically determinate** (internal forces can be calculated from balance with external forces) **or indeterminate** (for specification of internal forces we need deformation conditions in addition)?

- **Is it known what an accuracy of our calculation is?** Are results on a side of higher or lower safety? (see ex. in the part 6 „Thermal dilatations in HE“ or Guest x HMH x membrane hypotheses)

- **Will these calculations performed from designed operational parameters or from fault (extreme) conditions?**

What is an expected frequency of occurrence of these extreme conditions (ECs) and what are their maximal values?

<< → we can solve it with a safety coefficient normal (x = 1.5) or higher

→ we can solve it with a better process control, safety valves → elimination ECs

> → we must design such equipment on these extreme conditions

- **Are there internal stresses that arose during a part (equipment etc.) manufacturing** (welding, pressing, forming, machining, casting, jointing ...)? What are their values and what is their effect on internal stress (together with external forces)?

- **What are boundary conditions,** it is a way of equip. placing or join with other parts (supporting, hanging, way of fixation etc.)? It is very important parameter that has a big effect on results accuracy.

  (insufficiently compensated dilatations, equipment accidental overloading, inaccurately made piping and flanges...)

- **Are available data for solution of problems** of fatigue, creep, determination of safety factor or calculations according stochastic theory of operational reliability?

- **Is it possible to use (for more complicated cases) a finite elements method or some other computer program?**

C) Defining of equivalent stress

Calculated triaxial stress is compared with material strength that is a result from a tensile test based on the uniaxial stress.

Results must be comparable and reproducible. From tensile tests we obtain a start of shear of a tested bar for these characteristic parameters (yield point, strength limit etc.). On our real situation we can apply these parameters according following hypotheses:

- **Main (maximal) stress reaches yield point (σ_Y; $\sigma_{0,2}$)**

  Lamée ..... $\sigma_{emax} \leq \sigma_Y$ *(in one axis where is the stress maximal)*

- **Maximal shear stress reaches a limit of elastic shear of material** **providing that**

  Guest .... $\tau_{max} \leq \tau_Y = 0.5 * \sigma_Y$ *($\tau_{max} = \sigma_{max} / 2$)*

- **Tensile elongation ε_{amax} reaches a maximal limiting value ε_Y** **on the yield point, it is that**

  St. Venant .... $\varepsilon_{max} = \sigma_Y / E$ *(from Hooke´s law $\sigma = E * \varepsilon$)*

- Total energy needed for elongation that can a volume unit absorb reaches its maximal value on the yield point

$$\text{Haigh-Belrami} \ldots\ldots\ldots (E_{\Delta e})_{max} = 1/2 * \sigma_Y^2 / E$$

- Total energy needed for a shape change (elongation, compression, contraction ...) that can a volume unit absorb reaches its maximal value on the yield point

$$\text{HMH} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (E)_{max} = (1+\mu)/2 * \sigma_Y^2 / E$$

(Huber-Mises-Hencky – Poisson´s constant for steel is $\mu = 0{,}3$)

- Octohedral shear stress reaches its maximal value that is expressed in form

$$\tau_{max} = \sqrt{2/3} * \sigma_Y = 0.47 * \sigma_Y$$

Every of these 6 quantities forms a base for special hypothesis for specifying of an equivalent stress for uni-, bi- or triaxial stresses. Its proper choice has a big signification from the point of view of safety in operation and economical utilization of material. → simplification of a triaxial stress to a uniaxial value that will be compared with the allowed stress

D) Allowed stress

- This stress is given as a verified material characteristic given by tensile tests (e.g. the yield point) divided by the safety factor x.

- Safety factors are different for various engineering components, structures and equipment and are given by standards and regulations for their design, fabrication and operation.

For tension or compression stresses is the safety factor according Czech standards x = 1.5

For bending and other ways of plasticity use it can be only x = 1.0

For stability of beams, cylinders loaded with external pressure we use x = 2.4

- For common engineering calculations tabulated mechanical properties of materials are used together with given standard safety factors.

- For more exacting calculations we can take into account for example an effect of a change of material properties with time etc. (theory of reliability taking into account cumulative growth of damage, mathematical models of time change of material properties etc.).

  (e.g. for nuclear power plants design, where the very high reliability is necessary)

- Pressure vessels must be made from certified materials.

- In some cases is not a part dimensioned from a stress point of view but according technological needs (e.g. casts wall or a weldment thickness, tube plate thickness etc.).

  - A minimal wall thickness of a cast is specified from a point of view of a good melted iron running into all parts of a mould, iron cooling, mould production ....
  - A tube plate thickness is given not only from the stress point of view but mostly from the point of view of tubes beading = their fixation in the tube plate.

Why calculated values differ from designed (it is an equipment over- or undersizing)?

- Variability of strength characteristics of materials compared to tabulated average values resulting from tests → for more exact calculations (e.g. for pressure vessels) we must have certificates for every used material.
- Changes of mechanical properties in a part profile (for profiles with thicker walls is the difference higher than for thin ones → see example from material tables).
- Change of material strength depends on a character of its loading (e.g. speed of loading, material hardening or fatigue).
- Difference between calculated and actual working conditions of a structure, equipment etc. = worse working conditions (higher working pressure, temperature, problems with process control ...)
- Effect of other parts of a structure that are connected with the designed part etc. (rigidity or elasticity of a system and its parts, their relationships, relationship among forces etc.) - an additional stress in pipeline (insufficiently compensated thermal dilatations, not very precisely made pipeline ...

- Effects of stress raisers (concentrators) in fixations, joints, shape changes (footings, supports, necks, beams, holders etc.).

- Effect of additional forces caused by production technology or assembly (welding, pre-straining from assembly, insufficiently annealed cast = shrinkage stress etc.) - typical example is additional force (stress) on vessel necks from a bit shorter piping when are flanges drawn together with higher force of screws.

  (it is similar like it was in the previous part)

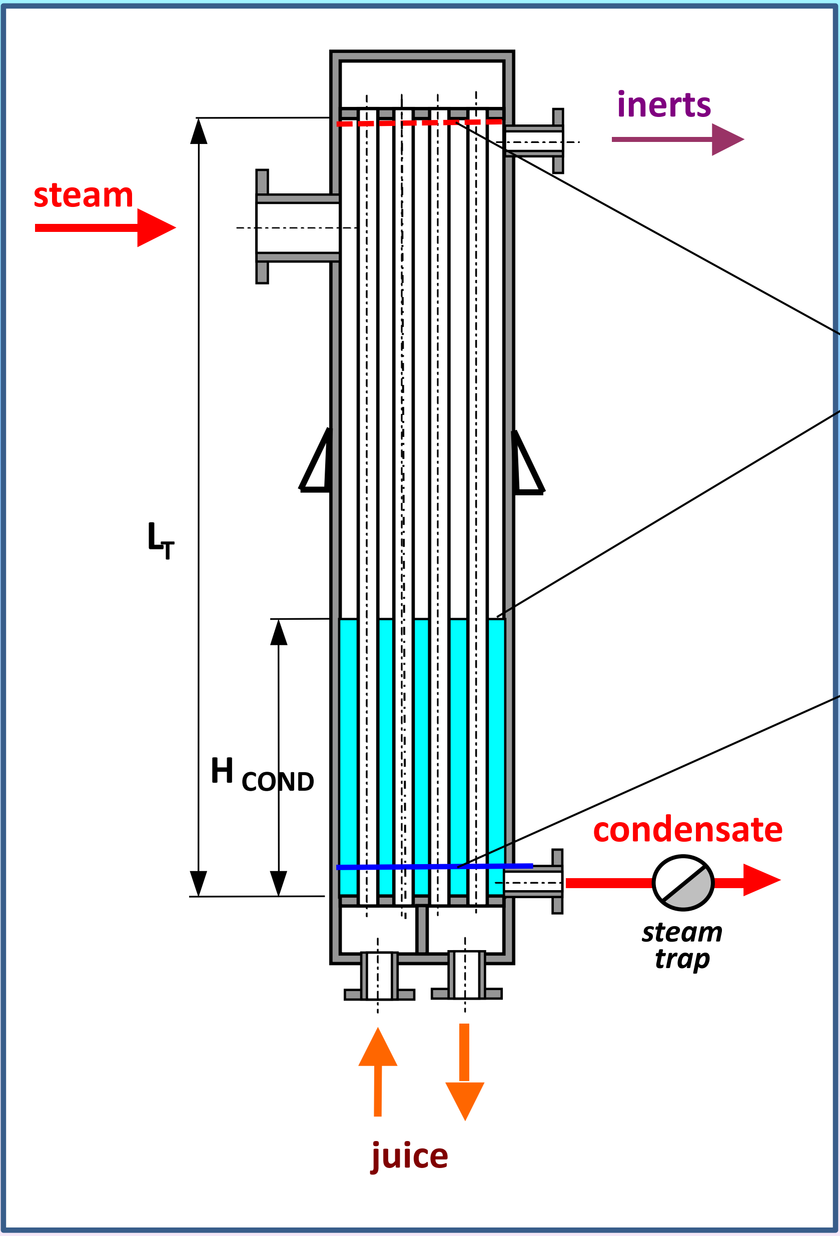
- Effect of overloading caused during operation by lack of technological discipline, problems with control, oper. troubles etc.

- Effect of internal stress caused by micro- and macro-roughness of surface, notches....

- Suitability or unsuitability of a hypothesis used for a case.

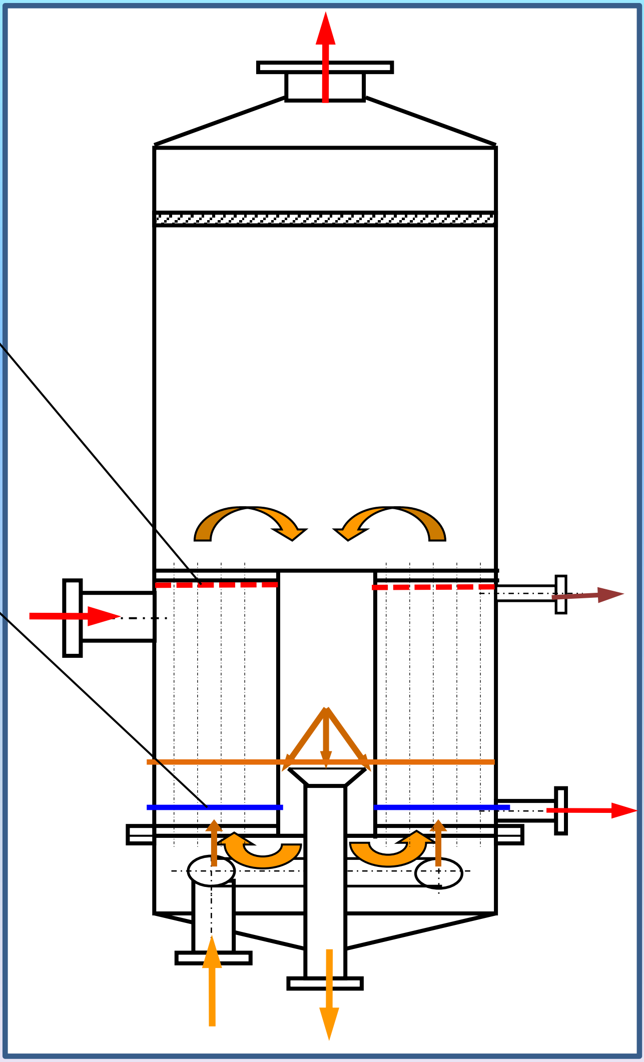
(A use of an inappropriate hypothesis is a designer mistake and this should not happen!)

Effect of bad condensate withdrawal



condensate level for bad working condensate drain

standard condensate level



Ex.: Real values from a sugar factory

Example of utilization of various hypothesis used for dimensioning of thick-walled cylindrical vessel loaded with internal overpressure

Given: r_i – internal radius; r_e – external radius; p_i – internal overpressure

$k = r_e / r_i = d_e / d_i$ – dimensionless wall thickness

The task usually is to specify following 3 internal pressures:

- **Internal pressure $(p_i)_{i=Y}$, at what starts to plasticize only some string but in the rest of the profile are only elastic deformations. A condition for this state is that the equivalent stress in the (e.g. external) string just reaches the yield point.**
- **If we transform pressure to a dimensionless quantity (p_i / σ_Y) we can obtain, for previously mentioned individual hypothesis, dependence of this dimensionless pressure (p_i / σ_Y) on the dimensionless wall thickness k (see fig. on p.71).**
- **Internal pressure $(p_i)_{e=Y}$ at what is material just plasticized in all profile (thickness). The condition is used for creep.**
- **Internal pressure $(p_i)_{i=P}$ at what comes to a material rupture etc. In some cases a material hardening is taken into account.**

Derivation of dependence $(p_i / \sigma_Y) - k$ for membrane theory of vessels (for cylinders)

(dimensionless pressure as function of dimensionless wall thickness)

Tangential (circumferential) stress is in a cylindrical vessel 2 x higher than an axial (see p. 74-76) – for sphere they are the same. Therefore we will take into account this stress. According a membrane theory in a membrane wall (thin-walled shell) are only tensile stresses (for internal pressure). The stress is the same in the all profile. We want to specify for what pressure the yield point is reached. $\sigma_{max} = \sigma_{tang} (\sigma_t)$

$$\sigma_t = p_i * r_i / s \leq \sigma_Y$$ where: $s = r_e - r_i$ $\quad k = r_e / r_i$ $\quad r_e = k * r_i$

After substitution and modification we obtain a membrane formula:

$$\sigma_t = p_i * r_i / \underbrace{(r_e - r_i)}_{s} = p_i * r_i / \underbrace{(k*r_i - r_i)}_{s} = p_i / (k-1) \leq \sigma_Y$$

$$(p_i / \sigma_Y) = k - 1$$

for $s = 0 \rightarrow k = 1 \rightarrow (p_i/\sigma_K) = 0 \rightarrow p_i = 0$

(= for this pressure and dimensionless wall thickness the tang. stress reaches the yield point)

Similar formulas can be derived for other hypotheses.

Lamée

$$(p_i / \sigma_Y) = (k^2 - 1) / (k^2 + 1)$$

(ASME for $k > 1.5$)

Dimensionless pressure as function of dimensionless wall thickness according various hypothesis

Guest

$$(p_i / \sigma_Y) = (k^2 - 1) / 2k^2$$

St. Venant

$$(p_i / \sigma_Y) = (k^2 - 1) / (1.3k^2 + 0{,}4)$$

H – B (Haigh – Belrami)

$$(p_i / \sigma_Y) = 2(k^2 - 1) / \sqrt{(6 + 10k^4)}$$

H-M-H (Huber – Mises – Hencky)

only for information

$$(p_i / \sigma_Y) = (k^2 - 1) / (\sqrt{3} * k^2)$$

ASME

$$(p_i / \sigma_Y) = (k - 1) / (0.6k + 0{,}4)$$

for $k \leq 1.5$

For fully plasticized state (profile):

only for information

Guest $(p_i / \sigma_Y) = \ln(k)$

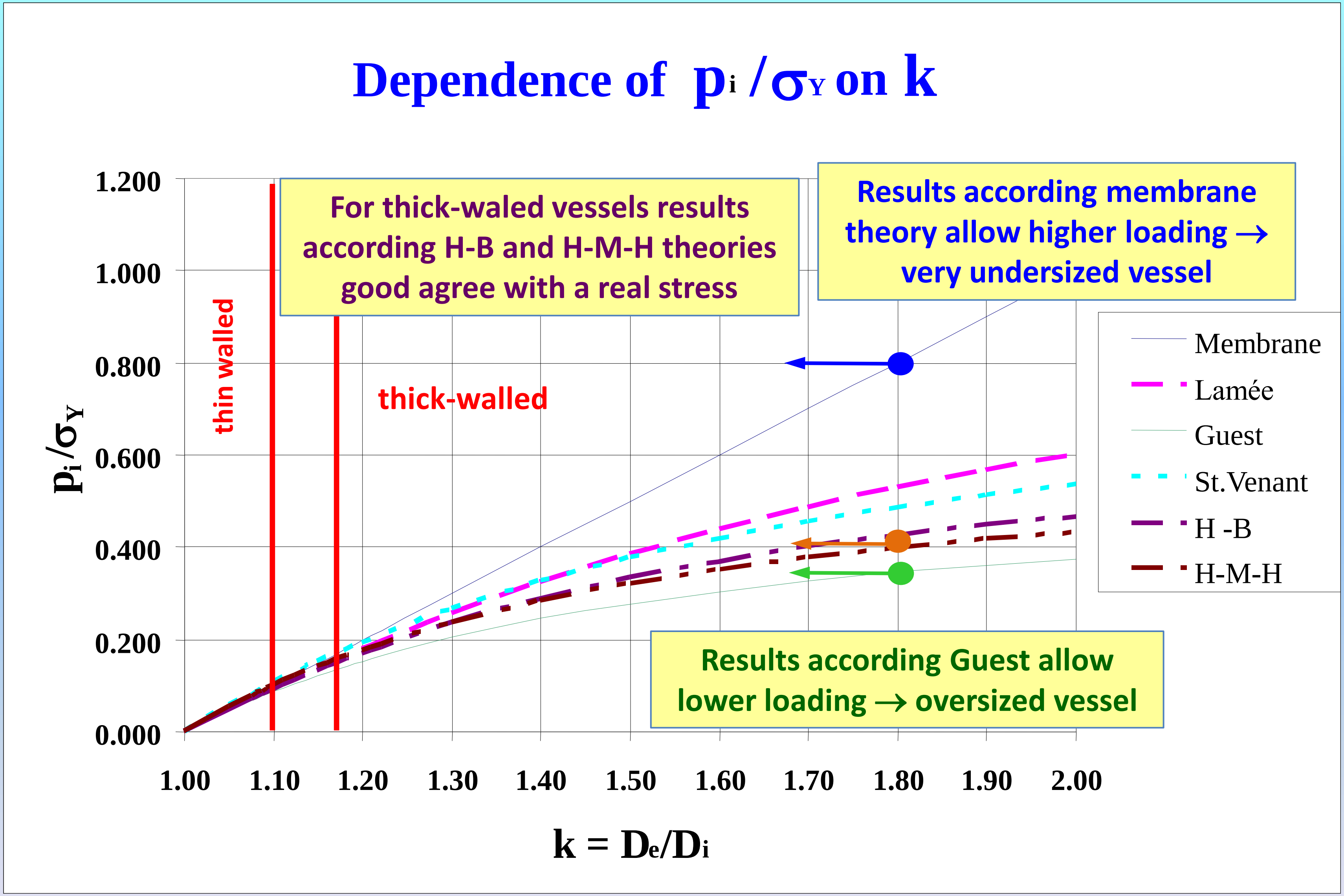
H-M-H $(p_i / \sigma_Y) = 2/\sqrt{3} * \ln(k) \approx 1.155 * \ln(k)$

Results calculated from these dependences are in the following table and on next figures.

Boundary between thin-walled and thick-walled cylinders is at value of $k = 1.17$. In practice more safety value $k = 1.1$ is used.

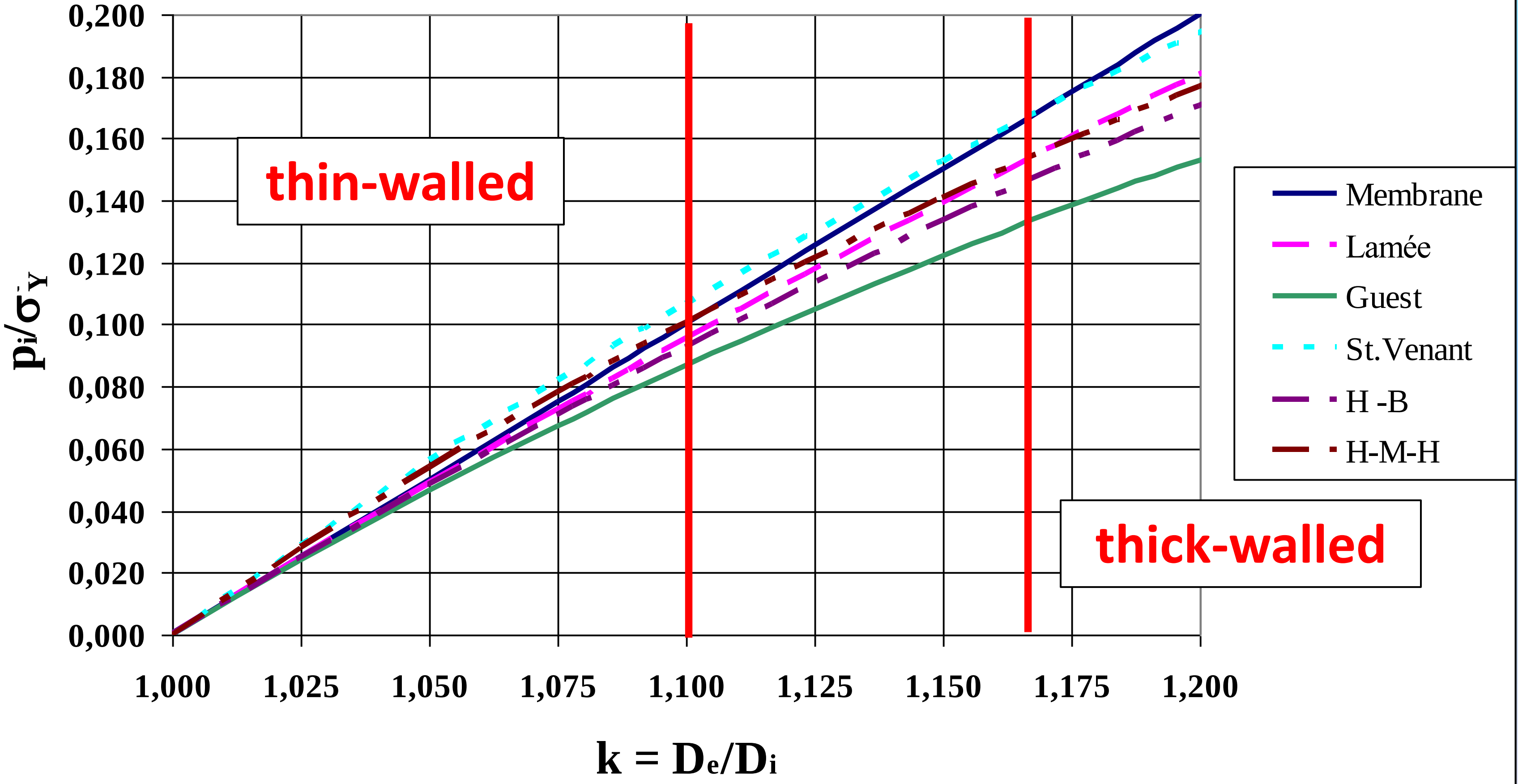
In the region ($k < 1.17$ – thin-walled cylinders) results according Guest are safer (they give lower allowed pressure) but the oversizing is not too high - see later.

Calculation according the membrane theory is very simple, therefore it is used very often for thin-walled cylinders ($k \leq 1.1$).



Dependence of maximal dimensionless pressure on dimensionless thickness of cylindrical vessel according various hypothesis.

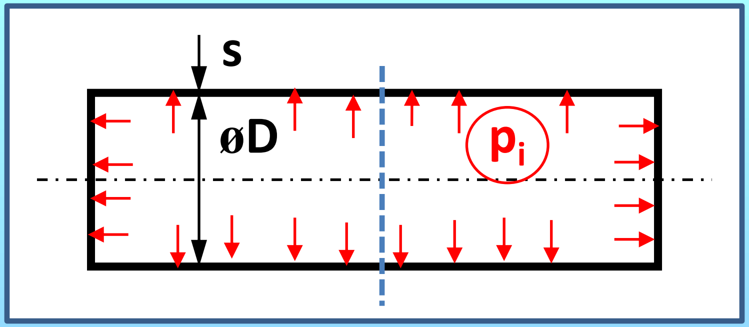
Dependence p_i/σ_Y on k



- In the region for k > 1.17 (thick-wall cylinders) results according hypothesis H-B and H-M-H are in the best conformity with actual measured values.
- Membrane formula allows much higher pressure and such cylinder would be very undersized!! Results according Lamée and St. Venant undersize a cylinder too, but not so much like the membrane theory.
- On the contrary results according Guest are somewhat oversized. (see previous figures and this table).

Theory used p_i/σ_Y	$k = 1.1$ *thin-wall*	$k = 1.17$ *theor. boundary*	$k = 1.4$ *thick-wall*	
Membrane (it permits higher p_i)	0.100	0.170 *+ 6 %*	0.400 !!!!	*+ 38 %*
Guest (it permits lower p_i)	0.087	0.157 *- 2 %*	0.245	*- 16 %*
Lameé	0.095	0.156 *- 3 %*	0.324	*+ 12 %*
St. Venant	0.106	0.169 *+5 %*	0.326	*+ 12 %*
H-B	0.092	0.148 *- 7 %*	0.288	*≈ 0 %*
H-M-H	0.100	0.156 *- 2 %*	0.283	*- 2 %*
ASME	0.094	0.154 *- 4 %*	0.323	*+ 11 %*
Correct values are	*≈ 0.100*	*≈ 0.160*	*≈ 0.290*	

9. Example:



Given:

Cylindrical vessel loaded with internal overpressure p_i = 0.6 MPa, with external diameter D_e = 1800 mm, material is steel with yield point σ_Y = 230 MPa.

Task:

What is a needed wall thickness of the vessel **s = ?.**

Because of simplification we will study a long cylinder without effects of stress peaks near covers and footings.

A) Specification of external loading

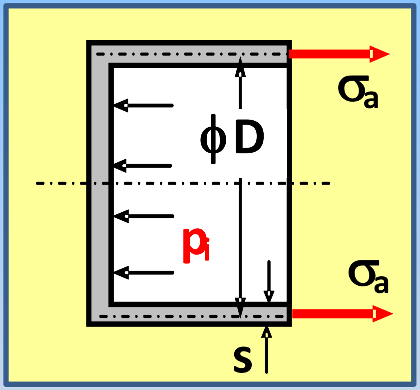
External loading (= internal overpressure) acts upright to the inner cylinder wall and causes internal stress in the cylinder wall in tangential, axial and radial directions. No other forces act on the vessel.

B) Calculation of internal stress

It is a typical statically determined structure with the membrane stress.

Only a balance of forces acting on the vessel is sufficient for calculation of the internal stresses. We can calculate only primary stresses as secondary stress and stress peaks are not in the vessel (we do not speak about covers etc.).

The balance of forces in the axial direction (a fictitious section with a plane upright on the cylinder axis)



external force $F_{ea} \approx \pi * D^2/4 * p_i$

(exactly for $D_i = D - s \approx D$)

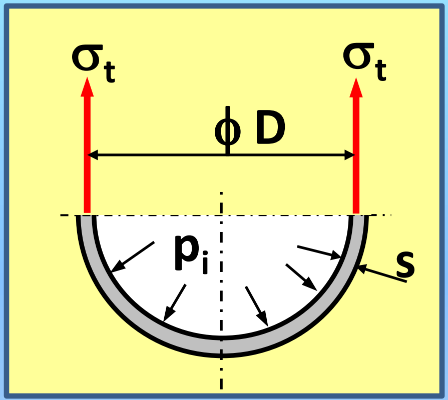
internal force $F_{ia} \approx \pi * D * s * \sigma_a$

- **Balance of forces in the section** $F_{ea} = F_{ia} \rightarrow$

$$\pi * D^2/4 * p_i \approx \pi * D * s * \sigma_a$$

- **Axial stress in the cylinder wall** $\sigma_a = p_i * D / 4s = p_i * r / 2s$

The balance of forces in the tangential direction
(a fictitious section with a plane in the cylinder axis)



Cylinder length is L

external force $F_{et} \approx p_i*D*L$

(exactly for $D_i = D - s \approx D$)

internal force $F_{it} = 2*L*s*\sigma_t$

- **Balance of forces in the section** $F_{et} = F_{it} \rightarrow$

(for thin-walled cylinders is the error small – Ex.)

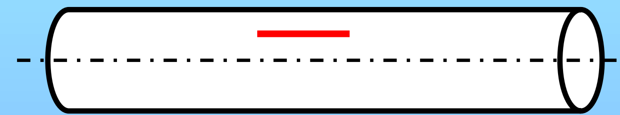
$$p_i*D*L* \approx 2*L*s*\sigma_t$$

- **Tangential stress in the cylinder wall**

$$\sigma_t = p_i * D / 2s = p_i * r / s$$

From it follows that the $\sigma_t = 2*\sigma_a$ $\rightarrow$ tubes and hoses rupture longitudinally!!!

The radial stress $\sigma_r = - p$



C) Definition of equivalent stress according Lamée and Guest hypothesis

For biaxial stress is according Lamée (σ_{max}) and Guest ($\tau_{max} = (\sigma_{max} - \sigma_{min})/2$) :

$$\tau_{max} = (\sigma_t - 0) / 2 \leq \tau_K = \sigma_Y / 2$$

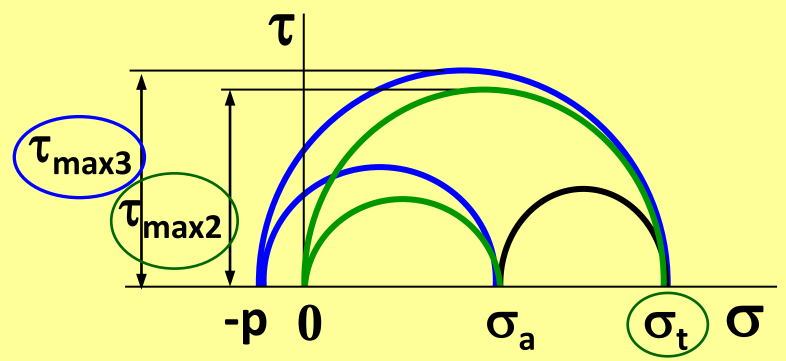


Fig.14: Mohr´s circles for the cylinder loaded with internal pressure

For both cases it is valid that $\sigma_e = \sigma_t$

Condition of dimensioning is $\sigma_t \leq \sigma_D$

For triaxial stress (we take into account radial (compression) stress in the cylinder wall too) following formulas are valid for definition of equivalent stress:

according Lamée $\sigma_e = \sigma_t \leq \sigma_Y$ $\sigma_e = \sigma_t \leq \sigma_D$

according Guest $\tau_{max} = (\sigma_t - (-p)) / 2 \leq \tau_K = \sigma_Y / 2$

$$\sigma_e = \sigma_t + p \leq \sigma_D$$

D) Allowed stress + condition of dimensioning

For the biaxial or triaxial stresses are in the wall only tensile and compression stresses (in the case of triaxial stress is the compression stress very small – see the next page) **and there are no bending moments. Therefore we must choose the safety factor x = 1.5 (see graph on p. 24). Allowed stress is thus**

$$\sigma_D = \sigma_Y / 1.5 = 230 / 1.5 = 153.3 \text{ MPa}$$

Needed calculated wall thickness (without effects of a weld weakening V, allowance for corrosion C, manufacturing tolerance e.g. 5±0.1 mm etc.) will be for various hypotheses:

***Biaxial stress according Lamée and Guest and triaxial stress according Lamée** (1st iteration for $D = D_e - s \approx D_e$)*

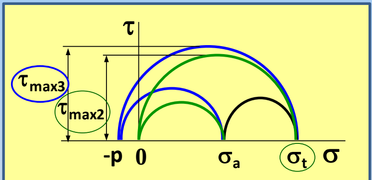


Fig.14: Mohr´s circles for the cylinder loaded with internal pressure

$$\boxed{\sigma_e = \sigma_t = p_i * r / s = p_i * D / 2s \leq \sigma_D}$$

$$s_{c1it} \geq p_i * D / (2 * \sigma_D) = 0.6*1800 / 2*153.3 = 3.52 \text{ mm}$$ *1.iter.* $D = D_e$

$$s_{c2it} \geq (0.6*(1800-3.52)) / 2*153.3 = 3.52 \text{ mm}$$ *2.iter.* $D_\phi = D_e - s$

Note: Calculation with an average diameter D_ϕ or external diameter D_e has not effect on the resulting value of calculated wall thickness (for our data = thin-walled cylinder).

Triaxial stress according Guest ($D = D_e - s \approx D_e$)

$\sigma_e = \sigma_t + p_i = p_i * r / s + p_i = p_i * D / 2s + p_i \leq \sigma_D$

$s_{c1it} \geq p_i * D / (2 * (\sigma_D - p_i))$ = 0.6*1800 / 2*(153.3 – 0.6) = 3.54 mm *(1.iter.)*

$s_{c2it} \geq (0.6*(\underbrace{1800 - 3.54}_{D_\phi})) / 2*(153.3 - 0.6) = 3.53$ mm *(2.iter.)*

According the Czech standards ČSN 690010

$s_c \geq p_i * D_e / (2 * \sigma_D * v - p_i) + c$

(for c = 0; v = 1 – see presumption above)

$s_c \geq p_i * D_e / (2 * \sigma_D - p_i) = 0.6*1800 / (2*153.3 - 0.6) = 3.53$ mm

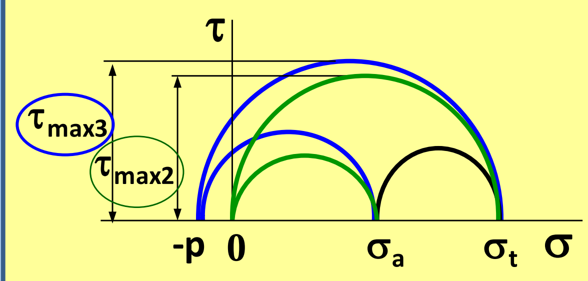


Fig.14: Mohr´s circles for the cylinder loaded with internal pressure

These individual results are from the point of view of technical praxis practically the same → for such thin-walled cylinder results do not depend on used theory.

Specification of single stresses in the cylindrical vessel wall

(for calculated wall thickness s_C = 3.53 mm)

- **Tangential stress**

$$\sigma_t = p_i * D / (2*s) = 0.6 * (1800 - 3.53) / (2 * 3.53) = 152.7 \text{ MPa}$$

- **Axial stress**

$$\sigma_a = \sigma_t / 2 = 152.7 / 2 = 76.3 \text{ MPa}$$

- **Radial stress**

$$\sigma_r = -p_i = -0.6 \text{ MPa}$$

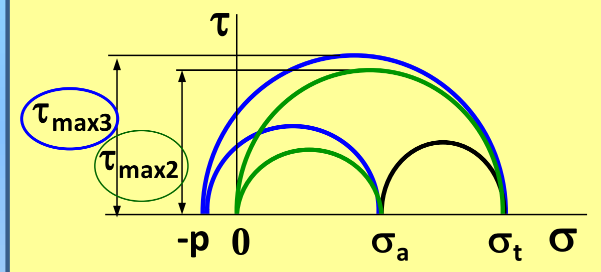


Fig.14: Mohr´s circles for the cylinder loaded with internal pressure

From this comparison follows that Mohr´s circles (fig.14) for τ_{max} are for the case of biaxial stress as well as for triaxial stress practically the same ($p_i <<< \sigma_a$ and σ_t).

The distance between points 0 and $-p_i$ is practically reduced to a point.

Use of previously derived dependences between dimensionless pressure (p_i/σ_Y) and dimensionless cylinder wall thickness ($k = d_e/d_i$) for the wall thickness calculations.

If we utilize relations derived before for these theories for equivalent stress specification (the diagrams on pages 71 and 72 are based on them) we can specify the minimal calculated wall thickness too. Instead the yield stress σ_Y we use the allowed stress σ_D. Results according various hypothesis are on the following pages.

Calculations are performed for 3 pressures so that we can see an effect of thin- or thick-walled cylinders.

Specification of cylinder wall thickness for given pressure according various hypothesis (see sooner derived formulas $p_i / s_K = f(k)$ and fig. on pages 71, 72)

$k = r_e / r_i = D_e / D_i = D_e / (D_e - 2s) \quad \rightarrow \quad s = D_e * (k - 1) / 2k$

Membrane theory

$(p_i / \sigma_Y) = k - 1 \quad \rightarrow \quad k = p_i / \sigma_D + 1$

D_e (mm)	σ_D (MPa)	p_i (MPa)	k (-)	s (mm)	
1800	153.3	0.6	1.00391	3.51	OK *(correctly is c. 3.5)*
1800	153.3	20.0	1.13046	103.9	a little undersized
1800	153.3	50.0	1.33	221.4	too undersized!!! *(correctly is c. 300 - 310)*

Lamée - σ_{max}

$(p_i / \sigma_Y) = (k^2 - 1) / (k^2 + 1) \quad \rightarrow \quad k = ((\sigma_D + p_i) / (\sigma_D - p_i))^{0,5}$

D_e (mm)	σ_D (MPa)	p_i (MPa)	k (-)	s (mm)	
1800	153.3	0.6	1.00392	3.52	thin-wall OK
1800	153.3	20.0	1.14021	110.7	≈ boundary
1800	153.3	50.0	1.40	258.5 !!	thick-wall – undersiz.

Guest - τ_{max}

$(p_i / \sigma_Y) = (k^2 - 1) / 2k^2 \quad \rightarrow \quad k = (\sigma_D / (\sigma_D - 2p_i))^{0,5}$

D_e (mm)	σ_D (MPa)	p_i (MPa)	k (-)	s (mm)	
1800	153.3	0.6	1.00394	3.53	OK
1800	153.3	20.0	1.1632	126.3	
1800	153.3	50.0	1.70	369	too oversized!! *(correctly is c. 300 - 310)*

St.Venant – ε_{max}

$(p_i / \sigma_Y) = (k^2 - 1) / (1.3k^2 + 0.4) \quad \rightarrow \quad k = ((s_D+0.4p_i) / (s_D-1.3p_i))^{0,5}$

D_e (mm)	σ_D (MPa)	p_i (MPa)	k (-)	s (mm)	
1800	153.3	0.6	1.00334	2.99	undersized *(c. 3.5)*
1800	153.3	20.0	1.1632	100.5	
1800	153.3	50.0	1.70	258	too undersized!! *(correctly c. 300 - 310)*

H-M-H - E_{max}

$(p_i / \sigma_Y) = (k^2 – 1) / (\sqrt{3}*k^2)$ → $k = (\sigma_D / (\sigma_D – 3^{0,5} * p_i))^{0,5}$

D_e (mm)	s_D (MPa)	p_i (MPa)	k (-)	s (mm)	
1800	153.3	0.6	1.00394	3.06	undersized *(c. 3.5)*
1800	153.3	20.0	1.1632	108.2	
1800	153.3	50.0	1.70	306	OK

On the next table is a comparison of all previous results for very thin-walled cylinders, cylinders on the boundary between thin- and thick-walled and thick-walled cylinders.

p_i (MPa)	Hypothesis (for k ≈ 1.004)				
0,6	Membrane	Lamée	Guest	St.Venant	H-M-H
s (mm)	3.51	3.52	3.53	2.99	3.06

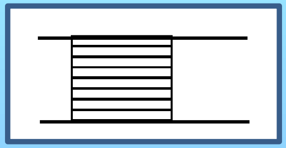
p_i (MPa)	Hypothesis (for k ≈ 1.16)				
20	Membrane	Lamée	Guest	St.Venant	H-M-H
s (mm)	103.9	110.7	126.3	100.5	108.2

p_i (MPa)	Hypothesis (for k ≈ 1.70)				
50	Membrane	Lamée	Guest	St.Venant	H-M-H
s (mm)	221.4	258.5	369	258	306

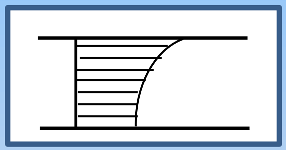
values that ≈ agree with a reality

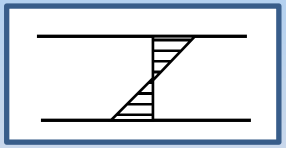
Some definitions:

Membranes: There are only tensile or compression forces (→ stresses). **Most common type of membranes = rotationally symmetrical membranes.** (spheres, cylinders, cones) **They are thin-walled, with constant stress in all profile section. They usually have continuous curvature, thickness and loading** (e.g. internal or external pressure).



Shells: They carry bending moments, torsion, shear, local forces etc. too. The stress is not uniform in a wall. **In a wall profile section are not only tensile or compression stresses but bending stress too (and the bending stress can vary** (see above – the elliptical tube).

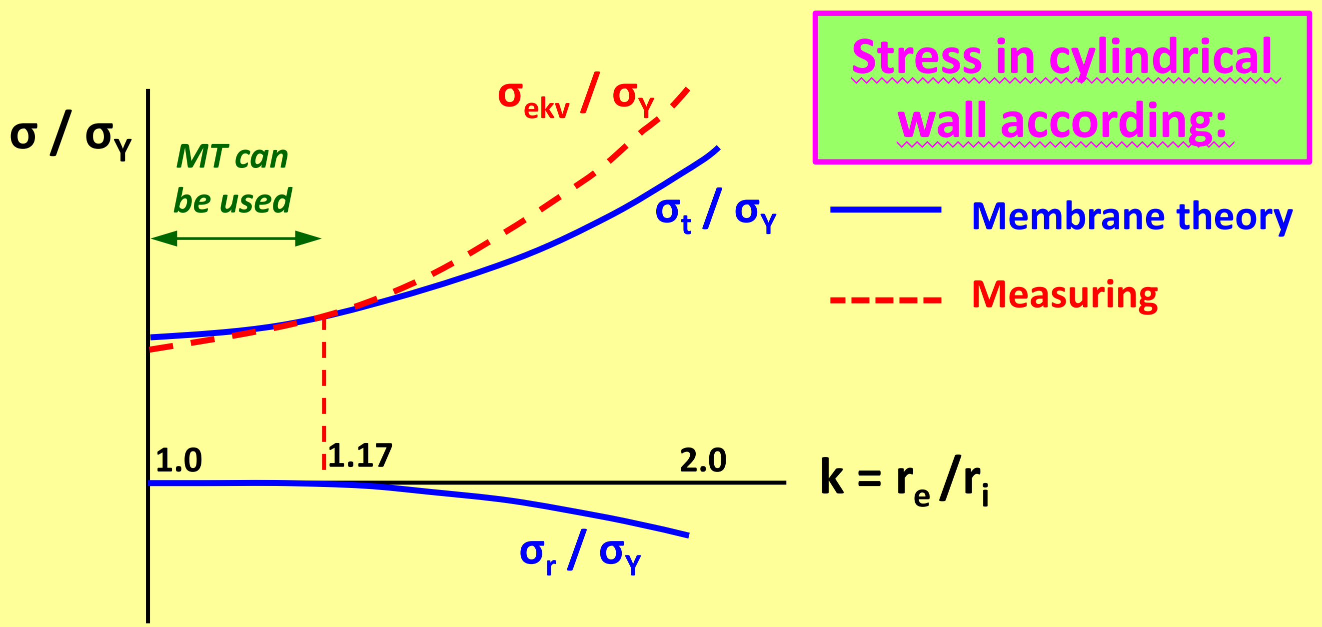




Loading: **Treated fluid pressure** (internal, external, hydrostatical).
Self-weight, weight of treated material.
Local forces and moments (supports, attachment, ...).
Wind, snow, seismicity.

Examples:

- Heat exchangers, evaporators.
- Vessels, tanks, silos.
- Reactors, columns, absorbers.
- Piping systems.



k < 1.17 → measured stress is < calculated → results are on the side of safety

k > 1.17 → measured stress is > calculated → undersized wall thickness

Laplace formula applied to membranes

$\Delta F = 0$

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = 0$$

Laplace general equation

$$N_\propto = \frac{\partial F}{\partial x} \qquad N_\beta = \frac{\partial F}{\partial y}$$

$$\frac{N_\alpha}{R_\beta} + \frac{N_\beta}{R_\alpha} + p_z = 0$$

$N_\alpha = \sigma_\alpha * s$

$N_\beta = \sigma_\beta * s$

$$\frac{N_\propto}{R_\beta} = \frac{\partial^2 F}{\partial x^2}$$

$$p = -\frac{\partial^2 F}{\partial z^2}$$

N_α [N/m] normal force in section for α = const. (angle between axis of rotation and section plane that cross the axis of rotation); the force is related to 1 m of membrane length (in the section) → it does not depend on a membrane thickness

N_β [N/m] normal force in section for β = const. (angle between chosen base plane and section plane that passes through axis of rotation)

R_α [m] radius of curvature of membrane in section for α = const.

R_β [m] radius of curvature of membrane in section for β = const.

Ex.1: Sphere, hemisphere – internal overpressure p

$R_\alpha = R_\beta = R$; $p_z = - p$

$$\frac{N_\alpha}{R} + \frac{N_\beta}{R} = p$$

$N_\alpha = N_\beta = N$

for sphere (axis can be chosen at will, forces must be the same) - it is symmetrical according its center

$$N = \frac{p * R}{2}$$

normal force in every section going by the sphere center

or for a membrane with wall thickness s is:

$$\sigma_\alpha = \sigma_\beta = \frac{N}{s} = \frac{p * R}{2 * s}$$

Ex.2: Cylinder – inner overpressure p, wall thickness s

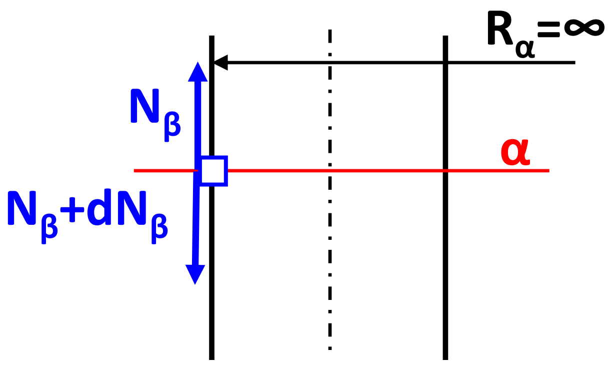
$R_\alpha = \infty$; $R_\beta = R$; $p_z = -p$

$$\frac{N_\alpha}{R_\beta} + \frac{N_\beta}{R_\alpha} = p = \frac{N_\alpha}{R} \longrightarrow N_\alpha = p * R$$

$$\sigma_\alpha = \sigma_t = \frac{N_\alpha}{s} = \frac{p * R}{s} = \frac{p * D}{2 * s}$$

N_β is not possible to specify from the Laplace formula $\rightarrow$ we can specify it for example from the forces balance in a cross section upright to the cylinder axis *(see above in this chapter. Example ad C)*

Section with plane β that passes through axis



Section with plane α that is upright axis